5.4 RAVNINSKO GIBANJE KRUTOG TIJELA

5.4.1 Jednadžbe gibanja krutog tijela

Za opisivanje gibanja krutog tijela u ravnini u x,y Descartesovom pravokutnom koordinatnom sustavu pogodno je koristiti Chalesov teorem:

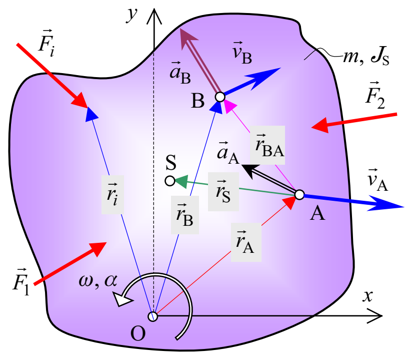
$$\vec{r}_{\mathrm{B}} = \vec{r}_{\mathrm{A}} + \vec{r}_{\mathrm{BA}} \tag{5.4.1}$$

te se nakon derivacije po vremenu može pisati:

$$\vec{v}_{\mathrm{B}} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}_{\mathrm{B}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{r}_{\mathrm{A}} + \vec{r}_{\mathrm{BA}}\right) \quad , \tag{5.4.2}$$

$$\vec{v}_{\mathrm{B}} = \dot{\vec{r}}_{\mathrm{A}} + \dot{\vec{r}}_{\mathrm{BA}} = \vec{v}_{\mathrm{A}} + \vec{\omega}\times\vec{r}_{\mathrm{BA}} \, , \tag{5.4.3}$$

$$\vec{v}_{\mathrm{B}} = \vec{v}_{\mathrm{A}} + \vec{v}_{\mathrm{BA}} \, . \tag{5.4.4}$$



Slika 5.4.1 *Uz definiciju ravninskog gibanja tijela*

Kada se sada derivira vektor brzine po vremenu može se izračunati vektor ubrzanja:

$$\vec{a}_{\mathrm{B}} = \frac{\mathrm{d}}{\mathrm{d}t}\vec{v}_{\mathrm{B}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{v}_{\mathrm{A}} + \vec{\omega}\times\vec{r}_{\mathrm{BA}}\right) , \tag{5.4.5}$$

$$\vec{a}_{\mathrm{B}} = \dot{\vec{v}}_{\mathrm{A}} + \dot{\vec{\omega}}\times\vec{r}_{\mathrm{BA}} + \vec{\omega}\times\dot{\vec{r}}_{\mathrm{BA}} = \vec{a}_{\mathrm{A}} + \vec{a}_{\mathrm{BA}} \, , \tag{5.4.6}$$

gdje su:

$$\vec{a}^{\mathrm{n}}_{\mathrm{BA}} = \vec{\omega}\times\dot{\vec{r}}_{\mathrm{BA}} = \vec{\omega}\times\vec{\omega}\times\vec{r}_{\mathrm{BA}} = -\omega^2\vec{r}_{\mathrm{BA}} \quad \mathrm{i} \tag{5.4.7}$$

$$\vec{a}^{\mathrm{t}}_{\mathrm{BA}} = \dot{\vec{\omega}}\times\vec{r}_{\mathrm{BA}} = \vec{\alpha}\times\vec{r}_{\mathrm{BA}} \, . \tag{5.4.8}$$

Neka je točka B proizvoljna opća točka tijela ($\vec{a}_{\mathrm{B}} = \vec{a}$ i $\vec{r}_{\mathrm{BA}} = \vec{r}$) te se primjenom II. Newtonovog zakona može pisati:

$$\int\limits_{(m)} \vec{a}\,\mathrm{d}m = \sum_{\mathrm{i}=1}^{\mathrm{n}} \vec{F}_{\mathrm{i}} = \vec{F}_{\mathrm{R}} \tag{5.4.9}$$

pa se nakon uvrštenja gornjih izraza za ubrzanje može pisati za bilo koju točku tijela:

$$\int\limits_{(m)} \vec{a}\,\mathrm{d}m = \vec{F}_{\mathrm{R}} = \int\limits_{(m)} \left(\vec{a}_{\mathrm{A}} + \vec{\alpha}\times\vec{r} - \omega^2\vec{r}\right)\mathrm{d}m \tag{5.4.10}$$

$$\int\limits_{(m)} \vec{a}\,\mathrm{d}m = \vec{F}_{\mathrm{R}} = \vec{a}_{\mathrm{A}} \int\limits_{(m)} \mathrm{d}m + \vec{\alpha}\times \int\limits_{(m)} \vec{r}\,\mathrm{d}m - \omega^2 \int\limits_{(m)} \vec{r}\,\mathrm{d}m \, . \tag{5.4.11}$$

Primjenom $\vec{r}_{\mathrm{S}} m = \int\limits_{(m)} \vec{r}\,\mathrm{d}m$ i $m = \int\limits_{(m)} \mathrm{d}m$ slijedi za translacijsko gibanje:

$$\vec{F}_{\mathrm{R}} = m\left(\vec{a}_{\mathrm{A}} + \vec{\alpha}\times\vec{r}_{\mathrm{S}} - \omega^2\vec{r}_{\mathrm{S}}\right) = m\vec{a}_{\mathrm{S}} \tag{5.4.12}$$

dakle:

$$\vec{F}_{\mathrm{R}} = m\vec{a}_{\mathrm{S}} \tag{5.4.13}$$