

DEFORMACIJE RAVNE GREDE OPTEREĆENE NA SAVIJANJE

1. OPTEREĆENJE KONCENTRIRANOM SILOM F

Zadano: F, l, a, c, EI_y ($b = l - a$, $a < l/2$)

a)

elast. linija

Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = -\frac{F a b}{6 l E I_y} (b + l), \quad \alpha_B = \frac{F a b}{6 l E I_y} (a + l),$$

$$\alpha_F = \alpha_B, \quad \alpha_C = -\frac{F a b}{3 l E I_y} (l - 2a),$$

$$\alpha_D = \frac{F a}{6 l E I_y} (l^2 - a^2 - 3c^2),$$

$$\alpha_E = \frac{F a}{24 l E I_y} (l^2 - 4a^2).$$

Progibi u karakterističnim presjecima grede:

$$w_C = \frac{F a^2 b^2}{3 l E I_y}, \quad w_D = \frac{F a c}{6 l E I_y} (l \cdot b + a \cdot b - c^2), \quad w_E = \frac{F a}{48 E I_y} (3l^2 - 4a^2), \quad w_F = -\alpha_B \cdot d = -\frac{F a d}{6 l E I_y} (l^2 - a^2).$$

b)

elast. linija

Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = \frac{F c l}{3 E I_y}, \quad \alpha_B = -\frac{F c l}{6 E I_y}, \quad \alpha_E = -\frac{F c l}{24 E I_y},$$

$$\alpha_C = \frac{F c}{6 E I_y} (2l + 3c), \quad \alpha_D = \frac{F c}{6 l E I_y} (l^2 - 3b^2).$$

Progibi u karakterističnim presjecima grede:

$$w_C = \frac{F c^2}{3 E I_y} (l + c), \quad w_D = -\frac{F c b}{6 l E I_y} (l^2 - b^2), \quad w_E = -\frac{F c l^2}{16 E I_y}.$$

c)

elast. linija

Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = \frac{F c l}{6 E I_y}, \quad \alpha_B = -\frac{F c l}{3 E I_y}, \quad \alpha_E = \frac{F c l}{24 E I_y},$$

$$\alpha_C = -\frac{F c}{6 E I_y} (2l + 3c), \quad \alpha_D = \frac{F c}{6 l E I_y} (l^2 - 3a^2).$$

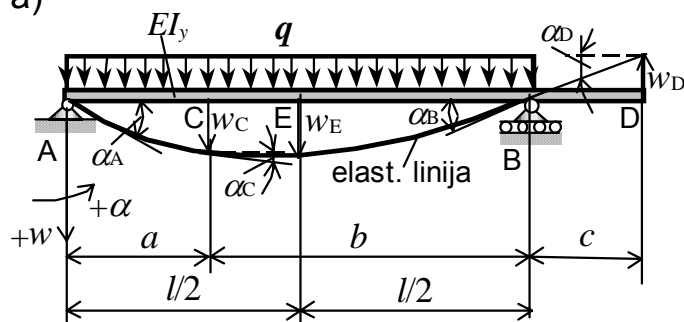
Progibi u karakterističnim presjecima grede:

$$w_C = \frac{F c^2}{3 E I_y} (l + c), \quad w_D = -\frac{F c a}{6 l E I_y} (l^2 - a^2), \quad w_E = -\frac{F c l^2}{16 E I_y}.$$

2. OPTEREĆENJE KONTUNIRANIM OPTEREĆENJEM q

Zadano: q, l, a, c, EI_y ($b = l - a$, $a < l/2$)

a)



Nagibi tangente na elastičnu liniju grede:

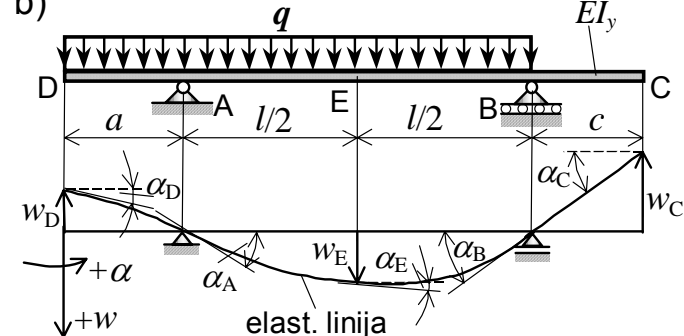
$$\alpha_A = -\frac{ql^3}{24EI_y}, \quad \alpha_B = \frac{ql^3}{24EI_y}, \quad \alpha_E = 0,$$

$$\alpha_C = -\frac{q}{24EI_y}(l^3 + 4a^3 - 6a^2l), \quad \alpha_D = \alpha_B.$$

Progibi u karakterističnim presjecima grede:

$$w_C = \frac{qa}{24EI_y}(l^3 - 2a^2l + a^3), \quad w_D = -\alpha_B \cdot c = -\frac{qcl^3}{24EI_y}, \quad w_E = \frac{5}{384} \frac{ql^4}{EI_y}.$$

b)



Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = -\frac{ql}{24EI_y}(l^2 - 4a^2), \quad \alpha_B = \frac{ql}{24EI_y}(l^2 - 2c^2),$$

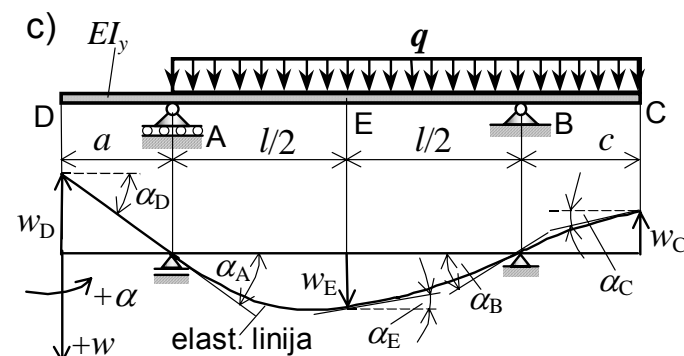
$$\alpha_C = \alpha_B, \quad \alpha_D = -\frac{q}{24EI_y}[l^3 - 4a^2(l + a)],$$

$$\alpha_E = -\frac{qla^2}{48EI_y}.$$

Progibi u karakterističnim presjecima grede:

$$w_C = -\alpha_B \cdot c = -\frac{qcl}{24EI_y}(l^2 - 2a^2), \quad w_D = -\frac{qa}{24EI_y}[l^3 - a^2(4l + 3a)], \quad w_E = \frac{ql^2}{384EI_y}(5l^2 - 12a^2).$$

c)



Nagibi tangente na elastičnu liniju grede:

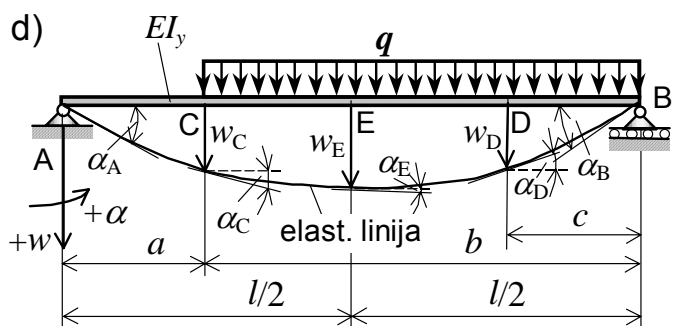
$$\alpha_A = -\frac{ql}{24EI_y}(l^2 - 2c^2), \quad \alpha_B = \frac{ql}{24EI_y}(l^2 - 4a^2),$$

$$\alpha_D = \alpha_A, \quad \alpha_C = \frac{q}{24EI_y}[l^3 - 4c^2(l + c)],$$

$$\alpha_E = \frac{qlc^2}{48EI_y}.$$

Progibi u karakterističnim presjecima grede:

$$w_D = -\alpha_A \cdot a = -\frac{qal}{24EI_y}(l^2 - 2c^2), \quad w_C = -\frac{qc}{24EI_y}[l^3 - c^2(4l + 3c)], \quad w_E = \frac{ql^2}{384EI_y}(5l^2 - 12c^2).$$



Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = -\frac{qb^2l}{12EI_y} \left[1 - \frac{1}{2} \left(\frac{b}{l} \right)^2 \right], \quad \alpha_B = \frac{qb^2l}{6EI_y} \left(1 - \frac{b}{2l} \right)^2,$$

$$\alpha_C = -\frac{qb^2l}{12EI_y} \left[1 - \frac{1}{2} \left(\frac{b}{l} \right)^2 - 3 \left(\frac{a}{l} \right)^2 \right],$$

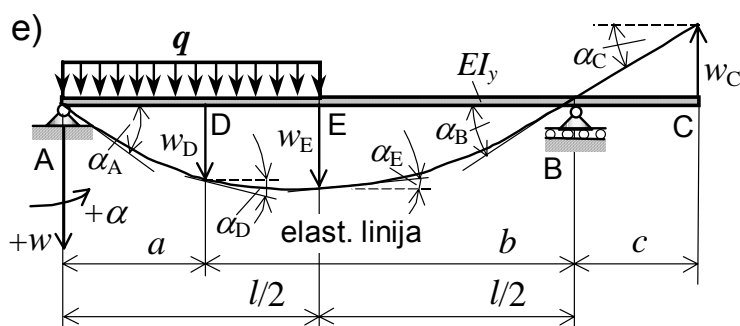
$$\alpha_D = -\frac{qb^2l}{12EI_y} \left[1 - \frac{1}{2} \left(\frac{b}{l} \right)^2 - 3 \left(\frac{l-c}{l} \right)^2 + \frac{2}{b^2l} (b-c)^3 \right],$$

$$\alpha_E = -\frac{qb^2l}{48EI_y} \left[1 - 2 \left(\frac{b}{l} \right)^2 + \frac{8}{b^2l} \left(\frac{l}{2} - a \right)^3 \right].$$

Progibi u karakterističnim presjecima grede:

$$w_C = \frac{qb^3l}{24EI_y} \left[4 - 7 \left(\frac{b}{l} \right) + 3 \left(\frac{b}{l} \right)^2 \right], \quad w_D = \frac{qb^3l}{24EI_y} \left[\left(1 + \frac{a}{l} \right)^2 \cdot \frac{c}{b} - 2 \frac{(l+a)c^3}{b^2l^2} + \frac{c^4}{b^3l} \right],$$

$$w_E = \frac{qb^2l^2}{48EI_y} \left[\left(1 + \frac{a}{l} \right)^2 - \frac{l+a}{2b} + \frac{1}{8} \left(\frac{l}{b} \right)^2 \right].$$



Nagibi tangente na elastičnu liniju grede:

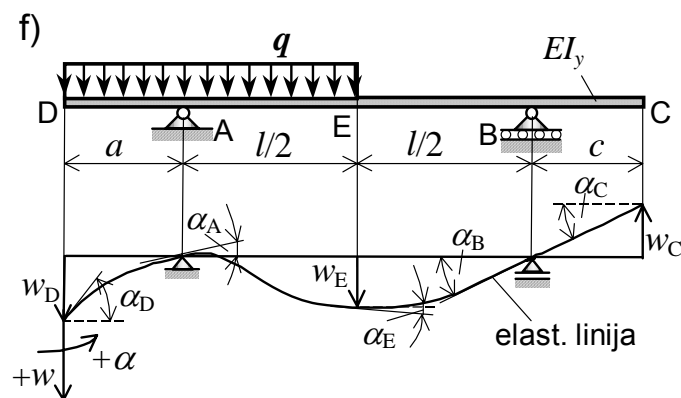
$$\alpha_A = -\frac{3}{128} \frac{ql^3}{EI_y}, \quad \alpha_B = \frac{7}{384} \frac{ql^3}{EI_y},$$

$$\alpha_C = \alpha_B, \quad \alpha_E = \frac{1}{384} \frac{ql^3}{EI_y},$$

$$\alpha_D = -\frac{1}{384} \frac{ql^3}{EI_y} \left[9 - 72 \left(\frac{a}{l} \right)^2 + 64 \left(\frac{a}{l} \right)^3 \right].$$

Progibi u karakterističnim presjecima grede:

$$w_C = -\alpha_B \cdot c = -\frac{7}{384} \frac{ql^3c}{EI_y}, \quad w_D = \frac{ql^3a}{192EI_y} \left[\frac{9}{2} - 12 \left(\frac{a}{l} \right)^2 + 8 \left(\frac{a}{l} \right)^3 \right], \quad w_E = \frac{5}{768} \frac{ql^4}{EI_y}.$$



Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = -\frac{ql^3}{384EI_y} \left[9 - 64 \left(\frac{a}{l} \right)^2 \right],$$

$$\alpha_B = \frac{ql^3}{384EI_y} \left[7 - 32 \left(\frac{a}{l} \right)^2 \right], \quad \alpha_C = \alpha_B,$$

$$\alpha_D = -\frac{ql^3}{384EI_y} \left[9 - 64 \left(\frac{a}{l} \right)^2 \cdot \left(1 + \frac{a}{l} \right) \right],$$

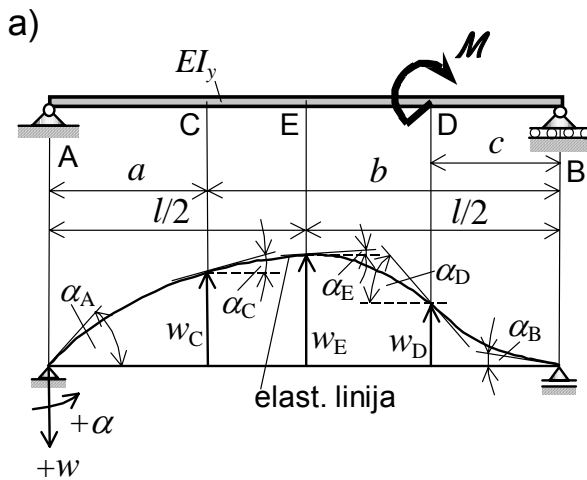
$$\alpha_E = \frac{ql^3}{384EI_y} \left[1 - 8 \left(\frac{a}{l} \right)^2 \right].$$

Progibi u karakterističnim presjecima grede na slici f):

$$w_C = -\alpha_B \cdot c = -\frac{ql^3}{384EI_y} \left[7 - 32 \left(\frac{a}{l} \right)^2 \right], \quad w_D = -\frac{ql^3}{384EI_y} \left[9 - 64 \left(\frac{a}{l} \right)^2 - 48 \left(\frac{a}{l} \right)^3 \right], \quad w_E = \frac{ql^2}{768EI_y} (5l^2 - 24a^2).$$

3. OPTEREĆENJE SPREGOM SILA M

Zadano: M, l, a, c, EI_y ($b = l - a, a < l/2$)



Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = \frac{Ml}{6EI_y} \left[1 - 3 \left(\frac{c}{l} \right)^2 \right], \quad \alpha_B = \frac{Ml}{6EI_y} \left[1 - 3 \left(1 - \frac{c}{l} \right)^2 \right],$$

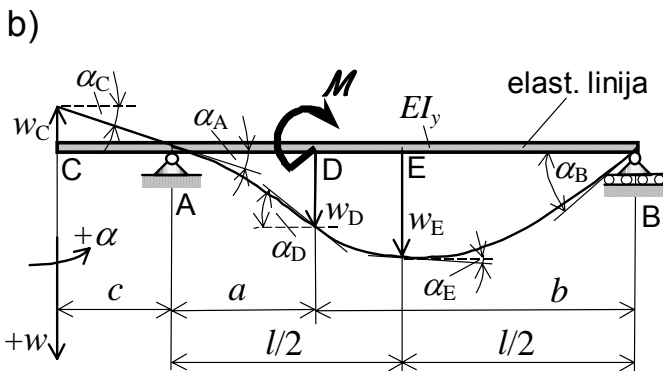
$$\alpha_C = \frac{Ml}{6EI_y} \left[1 - 3 \left(\frac{c}{l} \right)^2 - 3 \left(\frac{a}{l} \right)^2 \right],$$

$$\alpha_D = -\frac{Ml}{3EI_y} \left[1 - 3 \left(\frac{c}{l} \right) + 3 \left(\frac{c}{l} \right)^2 \right],$$

$$\alpha_E = \frac{Ml}{24EI_y} \left[1 - 12 \left(\frac{c}{l} \right)^2 \right].$$

Progibi u karakterističnim presjecima grede:

$$w_C = \frac{Mal}{6EI_y} \left[\left(\frac{a}{l} \right)^2 + 3 \left(\frac{c}{l} \right)^2 - 1 \right], \quad w_E = -\frac{Ml^2}{16EI_y} \left[1 - 4 \left(\frac{c}{l} \right)^2 \right], \quad w_D = \frac{Ml^2}{6EI_y} \left[\left(1 - \frac{c}{l} \right)^3 - \left[1 - 3 \left(\frac{c}{l} \right)^2 \right] \cdot \left(1 - \frac{c}{l} \right) \right].$$



Nagibi tangente na elastičnu liniju grede:

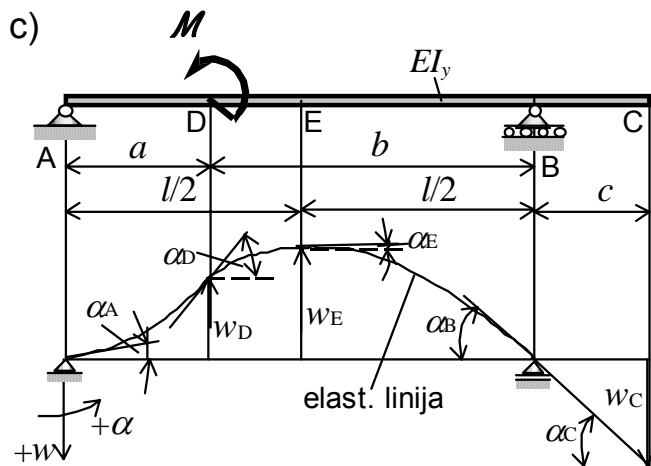
$$\alpha_A = \frac{Ml}{6EI_y} \left[1 - 3 \left(\frac{b}{l} \right)^2 \right], \quad \alpha_B = \frac{Ml}{6EI_y} \left[1 - 3 \left(\frac{a}{l} \right)^2 \right],$$

$$\alpha_C = \alpha_A, \quad \alpha_D = \frac{Ml}{6EI_y} \left[1 - 3 \left(\frac{a}{l} \right)^2 - 3 \left(\frac{b}{l} \right)^2 \right],$$

$$\alpha_E = \frac{Ml}{24EI_y} \left[1 - 12 \left(\frac{a}{l} \right)^2 \right].$$

Progibi u karakterističnim presjecima grede:

$$w_C = \alpha_A \cdot c = \frac{Mcl}{6EI_y} \left[1 - 3 \left(\frac{b}{l} \right)^2 \right], \quad w_D = \frac{Mal}{6EI_y} \left[\left(\frac{a}{l} \right)^2 + 3 \left(\frac{b}{l} \right)^2 - 1 \right], \quad w_E = \frac{Ml^2}{16EI_y} \left[1 - 4 \left(\frac{a}{l} \right)^2 \right].$$



Nagibi tangente na elastičnu liniju grede:

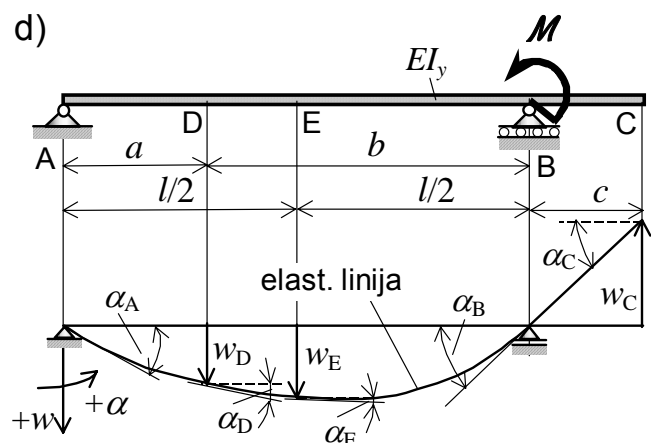
$$\alpha_A = \frac{Ml}{6EI_y} \left[3\left(\frac{b}{l}\right)^2 - 1 \right], \quad \alpha_B = \frac{Ml}{6EI_y} \left[3\left(\frac{a}{l}\right)^2 - 1 \right],$$

$$\alpha_C = \alpha_B, \quad \alpha_D = -\frac{Ml}{6EI_y} \left[1 - 3\left(\frac{a}{l}\right)^2 - 3\left(\frac{b}{l}\right)^2 \right],$$

$$\alpha_E = -\frac{Ml}{24EI_y} \left[1 - 12\left(\frac{a}{l}\right)^2 \right].$$

Progibi u karakterističnim presjecima grede:

$$w_C = -\alpha_B \cdot c = \frac{Mcl}{6EI_y} \left[1 - 3\left(\frac{a}{l}\right)^2 \right], \quad w_D = \frac{Mal}{6EI_y} \left[1 - 3\left(\frac{b}{l}\right)^2 - \left(\frac{a}{l}\right)^2 \right], \quad w_E = -\frac{Ml^2}{16EI_y} \left[1 - 4\left(\frac{a}{l}\right)^2 \right].$$



Nagibi tangente na elastičnu liniju grede:

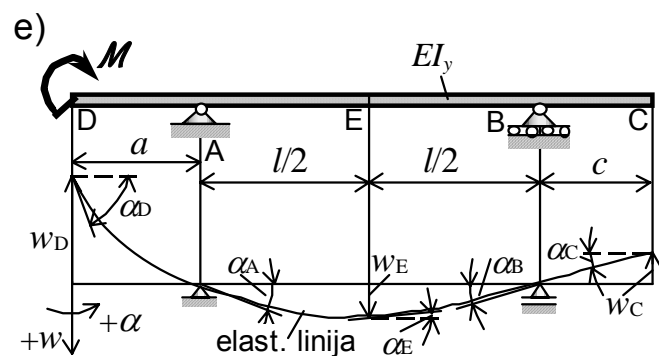
$$\alpha_A = -\frac{Ml}{6EI_y}, \quad \alpha_B = \frac{Ml}{3EI_y},$$

$$\alpha_C = \alpha_B, \quad \alpha_D = -\frac{Ml}{6EI_y} \left[1 - 3\left(\frac{a}{l}\right)^2 \right],$$

$$\alpha_E = -\frac{Ml}{24EI_y}.$$

Progibi u karakterističnim presjecima grede:

$$w_C = -\alpha_B \cdot c = -\frac{Mcl}{3EI_y}, \quad w_D = \frac{Mal}{6EI_y} \left[1 - \left(\frac{a}{l}\right)^2 \right], \quad w_E = \frac{Ml^2}{16EI_y}.$$



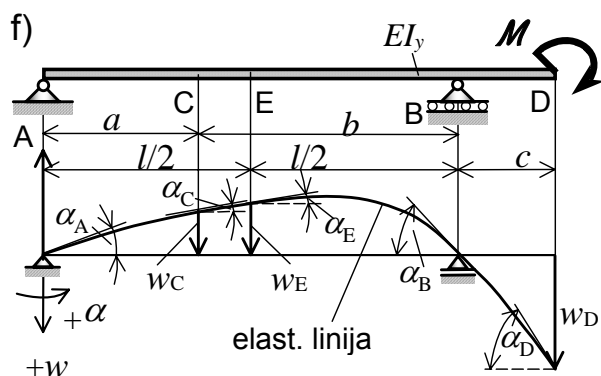
Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = -\frac{Ml}{3EI_y}, \quad \alpha_B = \frac{Ml}{6EI_y}, \quad \alpha_C = \alpha_B,$$

$$\alpha_D = -\frac{M}{3EI_y} (l + 3a), \quad \alpha_E = \frac{Ml}{24EI_y}.$$

Progibi u karakterističnim presjecima grede:

$$w_C = -\alpha_B \cdot c = -\frac{Mcl}{6EI_y}, \quad w_D = -\frac{Ma}{6EI_y} (2l + 3a), \quad w_E = \frac{Ml^2}{16EI_y}.$$



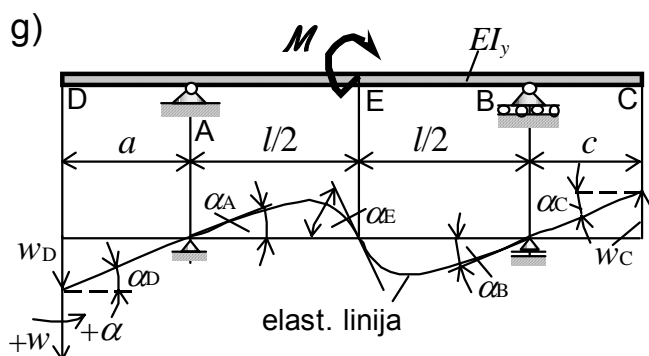
Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = \frac{Ml}{6EI_y}, \quad \alpha_B = -\frac{Ml}{3EI_y}, \quad \alpha_C = \frac{Ml}{6EI_y} \left[1 - 3 \left(\frac{a}{l} \right)^2 \right],$$

$$\alpha_D = -\frac{M}{3EI_y} (l + 3c), \quad \alpha_E = \frac{Ml}{24EI_y}.$$

Progibi u karakterističnim presjecima grede:

$$w_C = -\frac{M a l}{6EI_y} \left[1 - \left(\frac{a}{l} \right)^2 \right], \quad w_D = \frac{M c}{6EI_y} (2l + 3c), \quad w_E = -\frac{M l^2}{16EI_y}.$$



Nagibi tangente na elastičnu liniju grede:

$$\alpha_A = \alpha_B = \alpha_C = \alpha_D = \frac{Ml}{24EI_y},$$

$$\alpha_E = -\frac{Ml}{12EI_y}.$$

Progibi u karakterističnim presjecima grede:

$$w_C = -\alpha_B \cdot c = -\frac{M c l}{24EI_y}, \quad w_D = \alpha_A \cdot a = \frac{M a l}{24EI_y}, \quad w_E = 0.$$