

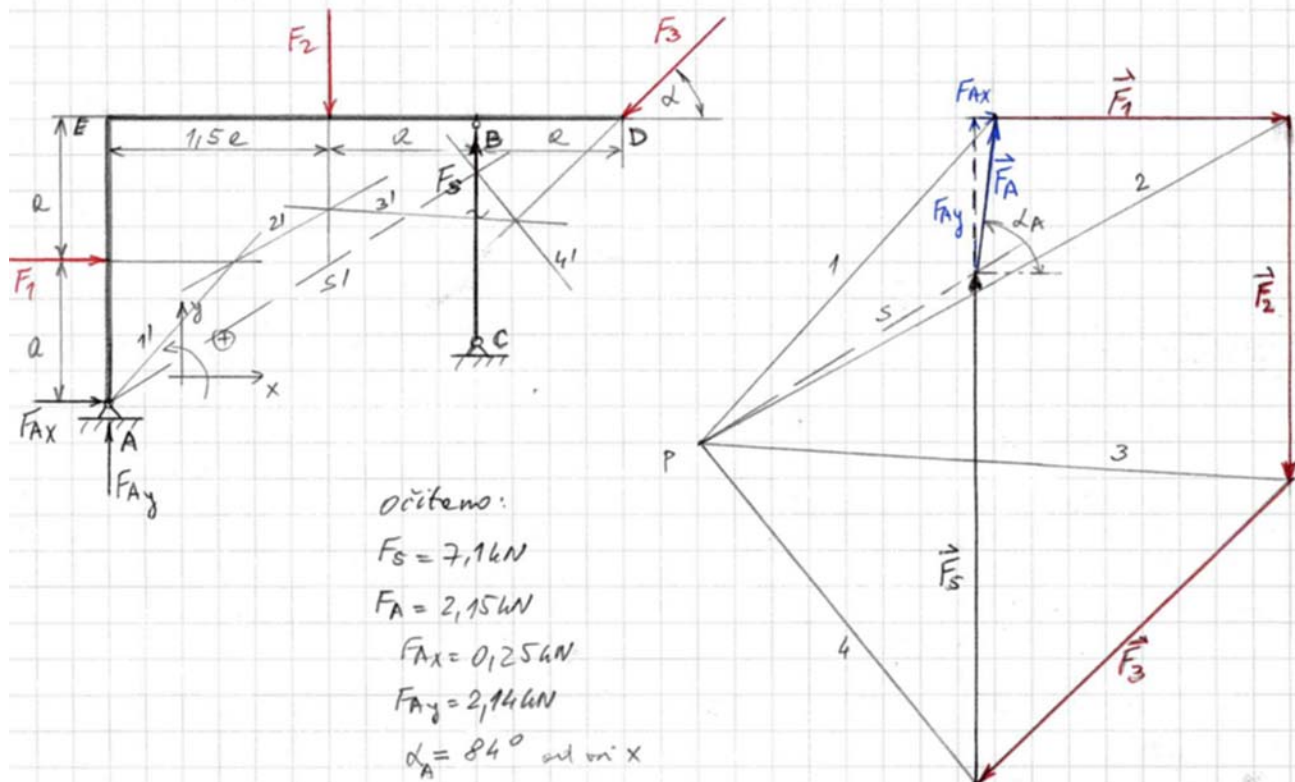
1. Štap AEBD bez težine nalazi se u uspravnoj ravnini, vezan je zglibno u nepomičnom osloncu A i sa štapom BC, prema slici. Kod opterećenja štapa silama  $F_1$ ,  $F_2$  i  $F_3$ , treba odrediti analitički i grafički (metoda verižnog poligona) silu u štapu BC i silu reakcije veze u osloncu A.

**Zadano:**  $a = 2 \text{ m}$ ,  $\alpha = 45^\circ$ ,  $F_1 = 4 \text{ kN}$ ,  $F_2 = 5 \text{ kN}$ ,  $F_3 = 6 \text{ kN}$ .

RJEŠENJE:

① Plan položaja:  $1 \text{ cm} \hat{=} 1 \text{ m}$

Plan sile:  $1 \text{ cm} \hat{=} 1 \text{ kN}$



Jedn. ravnoteže štapa ABDE:

$$1. \sum F_x = 0 \quad F_1 + F_{Ax} - F_3 \cdot \cos \alpha = 0$$

$$2. \sum F_y = 0 \quad F_{Ay} - F_2 - F_3 \cdot \sin \alpha + F_S = 0$$

$$3. \sum M_A = 0 \quad -F_1 \cdot a - F_2 \cdot 1,5a + F_S \cdot 2,5a - F_3 \cdot \sin \alpha \cdot 3,5a + F_3 \cdot \cos \alpha \cdot 2a = 0 / : a$$

$$2,5 F_S = F_1 + 1,5 F_2 + 3,5 \cdot \frac{1}{\sqrt{2}} F_3 - 2 F_3 \cdot \frac{1}{\sqrt{2}} = 4 + 1,5 \cdot 5 + 1,5 \cdot \frac{1}{\sqrt{2}} \cdot 6 = 17,865$$

$$F_S = \frac{17,865}{2,5} = \underline{\underline{7,146 \text{ kN}}} \quad \text{— sila u štapu BC}$$

$$F_{Ax} = -F_1 + F_3 \cdot \cos \alpha = -4 + 6 \cdot \frac{1}{\sqrt{2}} = -4 + 4,24264 = 0,24264 \text{ kN}$$

$$F_{Ay} = F_2 + F_3 \cdot \sin \alpha - F_S = 5 + 6 \cdot \frac{1}{\sqrt{2}} - 7,146 = 9,24264 - 7,146 \approx 2,097 \text{ kN}$$

$$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{0,24264^2 + 2,097^2} \approx \underline{\underline{2,111 \text{ kN}}} \quad \text{— reakcije veze u osloncu A}$$

$$\tan \alpha_A = \frac{F_{Ay}}{F_{Ax}} = \frac{2,097}{0,24264} = 8,641 \rightarrow \underline{\underline{\alpha_A \approx 83,4^\circ}} \quad \text{od osi } x \quad (\rightarrow)$$

2. Tereti težine  $Q_1$  i  $Q_2$  nalaze se na hrapavim podlogama i spojeni su zglobno sa štapom AB čija se težina zanemaruje, prema slici. Odrediti težinu tereta  $Q_1$  kod koje će sustav biti još u ravnoteži. Kolika je sila u štapu AB u tom slučaju? Zadatak riješiti analitički i grafički.

**Zadano:**  $\alpha = 30^\circ$ ,  $\mu_1 = 0,35$ ,  $\mu_2 = 0,25$ ,  $Q_2 = 500$  N.

**RJEŠENJE:**

②

$\tan \varphi_1 = \mu_1 = 0,35 \rightarrow \varphi_1 \cong 19,25^\circ$   
 $\tan \varphi_2 = \mu_2 = 0,25 \rightarrow \varphi_2 \cong 14,04^\circ$   
 Plus sila :  $1 \text{ cm} \cong 100 \text{ N}$

očitano:  
 $Q_1 = 740 \text{ N}$   
 $F_S = 375 \text{ N}$   
 $R_1 = 980 \text{ N}$   
 $R_2 = 450 \text{ N}$

$\beta = 90^\circ - (2\alpha + \varphi_2) = 15,86^\circ$

Ravnoteža tereta  $Q_2$ :

- $\sum F_{x_2} = 0 \quad Q_2 \cdot \cos(\alpha + \varphi_2) - F_S \cdot \cos \beta = 0$
- $\sum F_{y_2} = 0 \quad -Q_2 \cdot \sin(\alpha + \varphi_2) - F_S \cdot \sin \beta + R_2 = 0$

Sila u štapu AB:

$$F_S = Q_2 \cdot \frac{\cos(\alpha + \varphi_2)}{\cos \beta} = 500 \cdot \frac{0,71885}{0,96145} = \underline{\underline{373,84 \text{ N}}}$$

Reakcije podloge:

$$R_2 = Q_2 \cdot \sin(\alpha + \varphi_2) + F_S \cdot \sin \beta = 500 \cdot 0,6952 + 373,84 \cdot 0,275 = 347,58 + 102,8 = \underline{\underline{450,37 \text{ N}}}$$

Ravnoteža tereta  $Q_1$ :

- $\sum F_{x_1} = 0 \quad F_S \cdot \cos(\alpha + \varphi_1) - Q_1 \cdot \sin \varphi_1 = 0$
- $\sum F_{y_1} = 0 \quad R_1 - Q_1 \cdot \cos \varphi_1 - F_S \cdot \sin(\alpha + \varphi_1) = 0$

Težina tereta  $Q_1$ :

$$Q_1 = F_S \cdot \frac{\cos(\alpha + \varphi_1)}{\sin \varphi_1} = 373,84 \cdot \frac{0,65276}{0,3287} = \underline{\underline{740,17 \text{ N}}} \quad \text{— težina tereta } Q_1$$

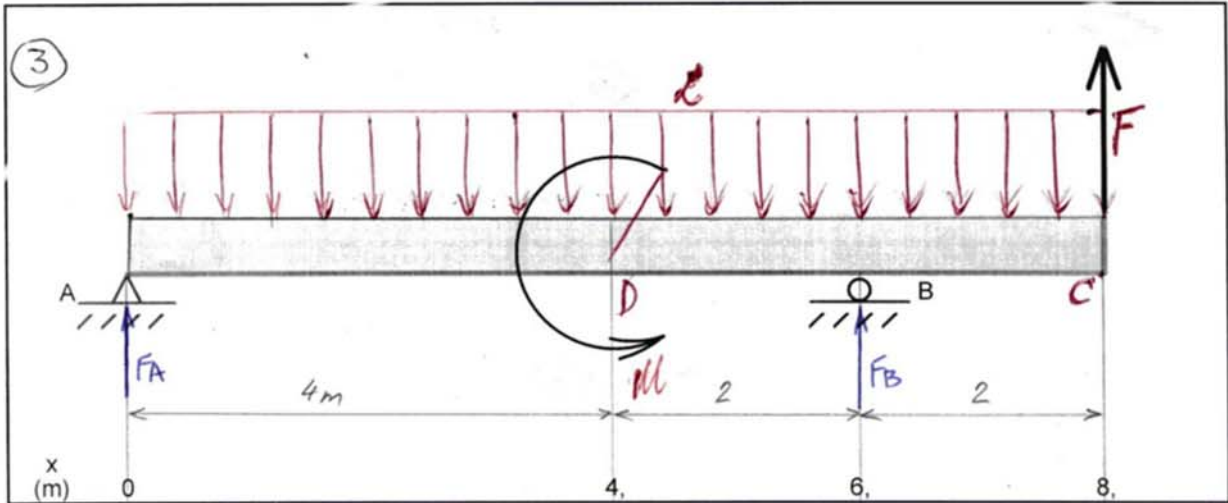
Reakcije podloge:

$$R_1 = Q_1 \cdot \cos \varphi_1 + F_S \cdot \sin(\alpha + \varphi_1) = 740,17 \cdot 0,9441 + 373,84 \cdot 0,75756 = 698,78 + 283,21 \cong \underline{\underline{982 \text{ N}}}$$

3. Ravni nosač ABCD zadan je i opterećen prema slici. Odrediti reakcije u osloncima A i B, te skicirati i kotirati dijagrame poprečnih sila i momenata savijanja.

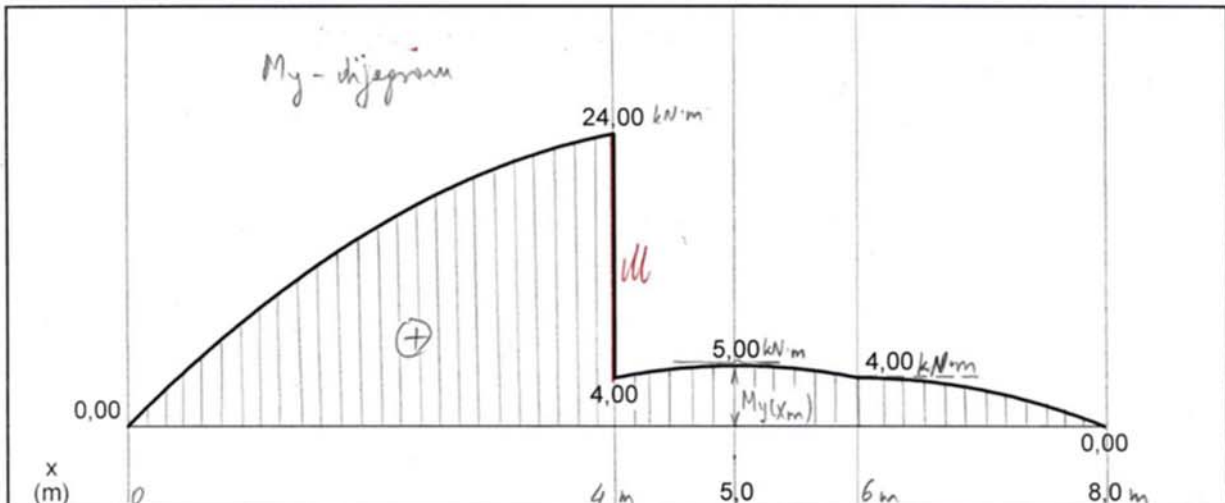
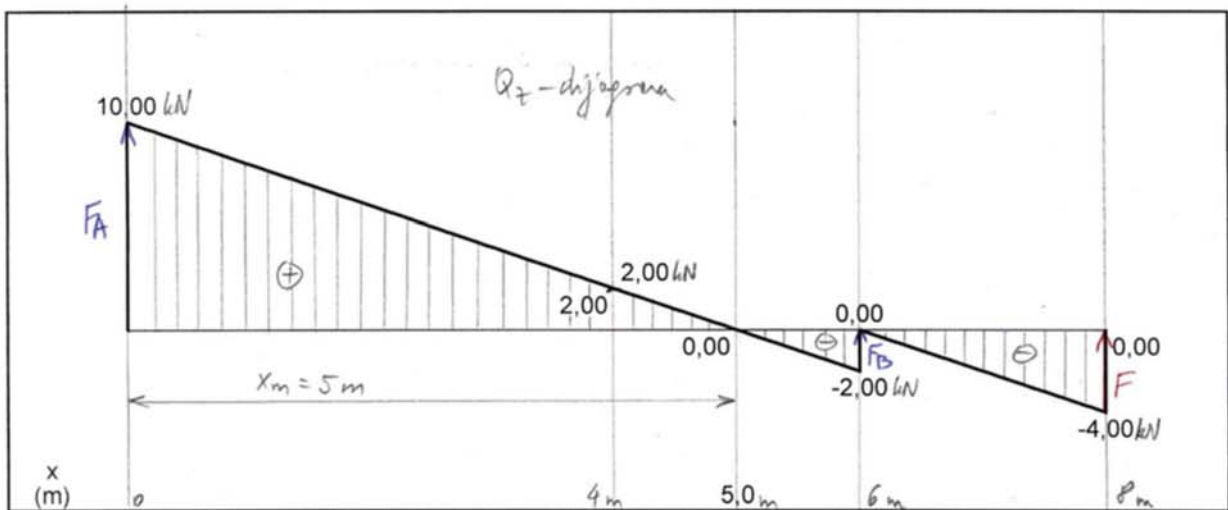
**Zadano:**  $a = 2 \text{ m}$ ,  $q = 2 \text{ kN/m}$ ,  $F = 4 \text{ kN}$ ,  $M = 20 \text{ kN}\cdot\text{m}$ .

RJEŠENJA: uz uporabu paketa programa „MDSolids“®



$F = 4,0 \text{ kN}$   
 $q = 2,0 \text{ kN/m}$   
 $M = 20,0 \text{ kN}\cdot\text{m}$

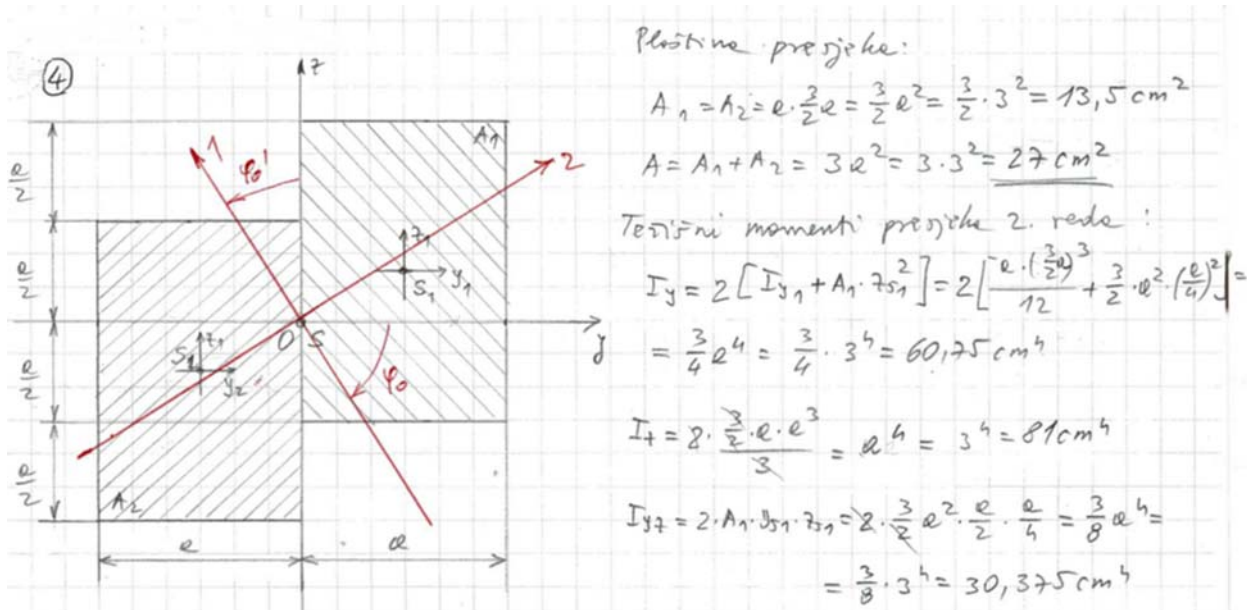
$F_A = 10,00 \text{ kN}$   
 $F_B = 2,00 \text{ kN}$



4. Za ravni presjek zadan prema slici odrediti glavne težišne momente presjeka 2. vrste ( $I_1$ ,  $I_2$ ) i glavne pravce presjeka 1, 2 ( $\varphi_0$ ), koje treba ucrtati na skici presjeka. Vrijednosti glavnih momenata presjeka 2. vrste treba kontrolirati grafički pomoću Mohrove kružnice.

**Zadano:**  $a = 3 \text{ cm}$ .

**RJEŠENJE:**



Glavni pravci momenta presjeka 2. vrste:

$$\tan 2\varphi_0' = -\frac{I_{yz}}{I_y - I_z} = -\frac{30,375}{60,75 - 81} = \frac{30,375}{10,125} = 3 \rightarrow 2\varphi_0' = 71,565^\circ \rightarrow \varphi_0' = 35,783^\circ \text{ od osi } y$$

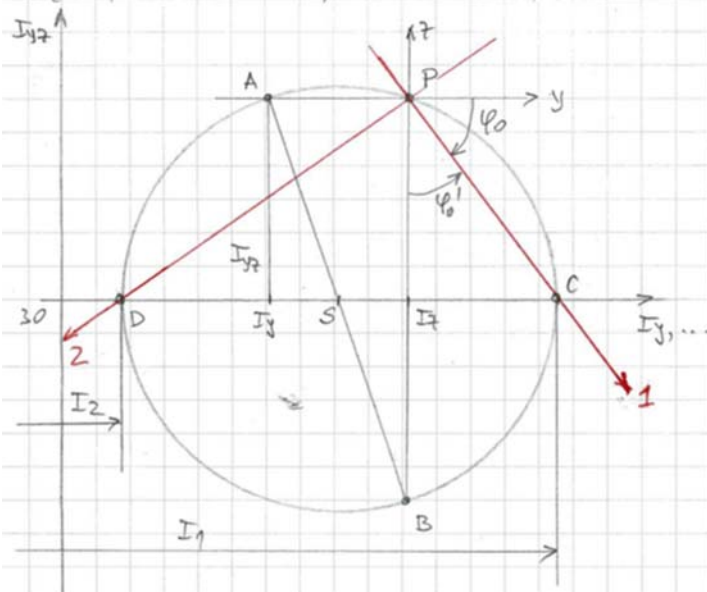
$$\varphi_0 = \varphi_0' - 90^\circ = -54,217^\circ \text{ od osi } y$$

Glavni težišni momenti presjeka 2. vrste:

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} = \frac{e^4}{8} (7 \pm \sqrt{10})$$

$$I_1 \approx 1,2703 \cdot e^4 = 1,2703 \cdot 3^4 \approx 102,893 \text{ cm}^4$$

$$I_2 \approx 0,4787 \cdot e^4 = 0,4787 \cdot 3^4 \approx 38,857 \text{ cm}^4$$



Očitano:

$$I_1 = 103 \text{ cm}^4$$

$$I_2 = 39 \text{ cm}^4$$

$$\varphi_0 = -55^\circ \text{ (od osi } y)$$

Polumjeri tromasti:

$$i_1 = \sqrt{\frac{I_1}{A}} = 1,95 \text{ cm}$$

$$i_2 = \sqrt{\frac{I_2}{A}} \approx 1,2 \text{ cm}$$