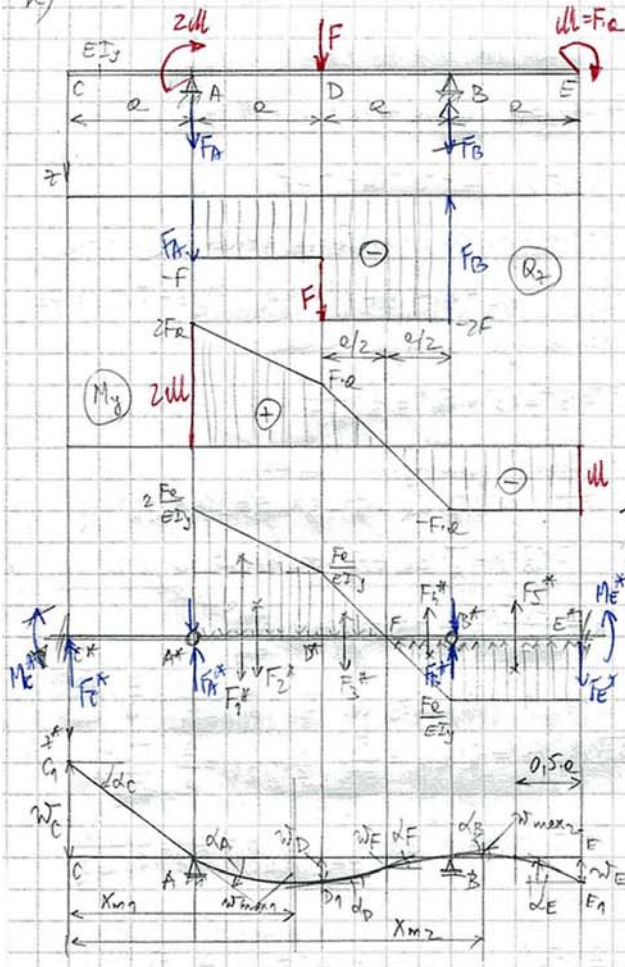
Primjer K: Deformacije ravnog nosača metodom analogne grede

Zadano: F , a , $\mathcal{M} = F \cdot a$, $EI_y = \text{konst.}$



Reakcije u oslonicima

1. $\Sigma F_z = 0 \Rightarrow \quad F_A + F_B + F = 0$

2. $\Sigma M_A = 0 \quad -2M + F \cdot a - M - F_B \cdot 2a = 0 \;/ : 2a$

$F_B = -2F \,, \quad F_A = -F - F_B = F$

$M_A = 2M = 2Fa, \quad M_D = 2M - F_A \cdot a = Fa, \quad M_E = M_B = -M = -Fa$

Opterećenje analogne grede:

$F_1^* = \frac{1}{2} \cdot \frac{Fa^2}{EI_y}$, $F_2^* = F_5^* = \frac{Fa^2}{EI_y}$, $F_3^* = F_4^* = \frac{1}{4} \cdot \frac{Fa^2}{EI_y}$

Reakcije analogne grede:

$\overline{AB^*}$: $1.\ \Sigma F_z^* = 0 \quad F_A^* + F_B^* = F_1^* + F_2^* + F_3^* - F_4^*$

$2.\ \Sigma M_A^* = 0 \quad F_B^* \cdot 2a = F_1^* \cdot \frac{a}{3} + F_2^* \cdot \frac{a}{2} - F_3^* \cdot \frac{2}{3}a \;/ : 2a$

$F_B^* = \frac{Fa^2}{2EI_y}\left(\frac{1}{2}\cdot\frac{1}{3} + 1\cdot\frac{1}{2} - \frac{1}{4}\cdot\frac{2}{3}\right) = \frac{1}{4}\cdot\frac{Fa^2}{EI_y}$

$F_A^* = F_1^* + F_2^* - F_B^* = \frac{Fa^2}{EI_y}\left(\frac{1}{2} + 1 - \frac{1}{4}\right) = \frac{5}{4}\cdot\frac{Fa^2}{EI_y}$

$\overline{AC^*}$: $F_C^* = F_A^* = \frac{5}{4}\cdot\frac{Fa^2}{EI_y}$, $M_C^* = -F_A^* \cdot a = -\frac{5}{4}\cdot\frac{Fa^3}{EI_y}$

$\overline{BE^*}$: $F_E^* = F_5^* - F_B^* = \frac{Fa^2}{EI_y}\left(1 - \frac{1}{4}\right) = \frac{3}{4}\cdot\frac{Fa^2}{EI_y}$

$M_E^* = F_5^* \cdot \frac{a}{2} - F_B^* \cdot a = \frac{Fa^3}{EI_y}\left(1\cdot\frac{1}{2} - \frac{1}{4}\cdot 1\right) = \frac{1}{4}\cdot\frac{Fa^3}{EI_y}$

Nagibi tangente na elastičnu linij:

$\alpha_A = -Q_A^* = -F_A^* = -\frac{5}{4}\cdot\frac{Fa^2}{EI_y}$, $\alpha_B = -Q_B^* = F_B^* = \frac{1}{4}\cdot\frac{Fa^2}{EI_y}$, $\alpha_C = -Q_C^* = -F_C^* = -\frac{5}{4}\cdot\frac{Fa^2}{EI_y}$

$-1{,}1250$ $\quad 0{,}1250$ $\quad -1{,}1250$

$\alpha_D = -Q_D^* = F_B^* - F_3^* + F_4^* = \frac{1}{4}\cdot\frac{Fa^2}{EI_y}$, $\alpha_E = -Q_E^* = -F_E^* = -\frac{3}{4}\cdot\frac{Fa^2}{EI_y}$, $\alpha_F = -Q_F^* = F_B^* + F_4^* = \frac{1}{2}\cdot\frac{Fa^2}{EI_y}$

$0{,}1250$ $\quad -0{,}1250$ $\quad 0{,}5000$

Progibi grede: $w_A = 0$, $w_B = 0$

$w_C = M_C^* = -\frac{5}{4}\cdot\frac{Fa^3}{EI_y}$, $w_E = M_E^* = \frac{1}{4}\cdot\frac{Fa^3}{EI_y}$

$-1{,}1250$ $\quad 0{,}1250$

$w_D = M_D^* = F_B^* \cdot a + F_4^* \cdot \frac{2}{3}a = \frac{Fa^3}{EI_y}\left(\frac{1}{4}\cdot 1 + \frac{1}{4}\cdot\frac{2}{3}\right) = \frac{5}{12}\cdot\frac{Fa^3}{EI_y}$

$0{,}416667$

$w_F = M_F^* = F_B^* \cdot \frac{a}{2} + F_4^* \cdot \frac{a}{3} = \frac{Fa^3}{EI_y}\left(\frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{3}\right) = \frac{5}{24}\cdot\frac{Fa^3}{EI_y}$

$0{,}2083333$

$x_{m1} = 1{,}1753 \cdot a$, $w_{max_1} = 0{,}1145706 \cdot \frac{Fa^3}{EI_y}$

$x_{m2} = 3{,}250 \cdot a$, $w_{max_2} = -0{,}031250 \cdot \frac{Fa^3}{EI_y}$