

Primjer I: Deformacije ravnog nosača metodom analogne grede

Zadano: $F, a, M = F \cdot a, EI_y = \text{konst.}$

I)

Reakcije u obojicama:

$$1. \sum F_z = 0 \Rightarrow F + F_A + F_B = 0$$

$$2. \sum M_A = 0 \Rightarrow F \cdot a - M + M - F_B \cdot 2a = 0$$

$$F_B = \frac{M}{2a} = \frac{F \cdot a}{2}, F_A = -F = F_B = -\frac{3}{2} F$$

$M_{A,C} = -F \cdot a, M_{A,D} = M_{A,C} + M = 0, M_B = M = F \cdot a$

Opterećenje analogne grede:

$$F_1^* = \frac{1}{2} \cdot \frac{F \cdot a^2}{EI_y}, F_2^* = \frac{F \cdot a^2}{EI_y}, F^* = \frac{1}{4} \cdot \frac{F \cdot a^2}{EI_y}$$

Reakcije analogne grede:

$$\overline{AB}^*: 1. \sum F_z^* = 0 \Rightarrow F_A^* + F_B^* = F_2^*$$

$$2. \sum M_A^* = 0 \Rightarrow F_B^* \cdot 2a - F_2^* \cdot \frac{4}{3} a = 0 \Rightarrow F_B^* = \frac{2}{3} F_2^* = \frac{2}{3} \cdot \frac{F \cdot a^2}{EI_y}, F_A^* = \frac{1}{3} F_2^* = \frac{1}{3} \cdot \frac{F \cdot a^2}{EI_y}$$

$\overline{AC}^*: F_C^* = F_A^* - F_1^* = \frac{F \cdot a^2}{EI_y} \left(\frac{1}{3} - \frac{1}{2} \right) = -\frac{1}{6} \cdot \frac{F \cdot a^2}{EI_y}$

$M_C^* = F_C^* \cdot \frac{2}{3} a - F_A^* \cdot a = \frac{F \cdot a^3}{EI_y} \left(\frac{1}{2} \cdot \frac{2}{3} - \frac{1}{3} \cdot 1 \right) = 0$

$\overline{BE}^*: F_E^* = F_B^* = \frac{2}{3} \cdot \frac{F \cdot a^2}{EI_y}, M_E^* = -F_B^* \cdot a = -\frac{2}{3} \cdot \frac{F \cdot a^3}{EI_y}$

Način tangente na elastičnu liniju: $\alpha_i = -Q_i^*$

$$\alpha_A = -Q_A^* = -F_A^* = -\frac{1}{3} \cdot \frac{F \cdot a^2}{EI_y}, \alpha_B = -Q_B^* = F_B^* = \frac{2}{3} \cdot \frac{F \cdot a^2}{EI_y}, \alpha_C = -Q_C^* = +F_C^* = \frac{1}{6} \cdot \frac{F \cdot a^2}{EI_y}$$

$$\alpha_D = -Q_D^* = -F_A^* + F^* = \frac{F \cdot a^2}{EI_y} \left(-\frac{1}{3} + \frac{1}{4} \right) = -\frac{1}{12} \cdot \frac{F \cdot a^2}{EI_y}$$

Prigibi grede: $w_i = M_i^*$

$$w_C = M_C^* = 0, w_E = M_E^* = -\frac{2}{3} \cdot \frac{F \cdot a^3}{EI_y}, w_D = M_D^* = F_A^* \cdot a - F^* \cdot \frac{a}{3} = \frac{F \cdot a^3}{EI_y} \left(\frac{1}{3} \cdot 1 - \frac{1}{3} \cdot \frac{1}{3} \right) = \frac{1}{4} \cdot \frac{F \cdot a^3}{EI_y}$$

Ekstremne vrijednosti prigiba:

$$F^* = F_C^* = \frac{1}{6} \cdot \frac{F \cdot a^2}{EI_y}, y_1 = \frac{x_1}{a}, F^* = \frac{1}{2} \cdot x_1 \cdot y_1 = \frac{x_1^2}{2a} \cdot \frac{1}{a} \Rightarrow x_1^2 = \frac{a^2}{3} \Rightarrow x_1 = \frac{a}{\sqrt{3}} \approx 0,57735a$$

$$w(x_1) = -F_C^* \cdot x_1 + F^* \cdot \frac{x_1}{3} = \frac{F \cdot a^3}{EI_y} \left(-\frac{1}{6} \cdot \frac{1}{\sqrt{3}} + \frac{1}{6} \cdot \frac{1}{3\sqrt{3}} \right) = -\frac{F \cdot a^3}{9\sqrt{3} EI_y} \approx -0,06215 \cdot \frac{F \cdot a^3}{EI_y}$$

$$F^* = F_A^* = \frac{1}{3} \cdot \frac{F \cdot a^2}{EI_y}, y_2 = \frac{x_2'}{2a}, F^* = \frac{1}{2} \cdot x_2' \cdot y_2 = \frac{1}{4} \cdot \frac{x_2'^2}{2a} = \frac{1}{8} \cdot \frac{x_2'^2}{a} \Rightarrow x_2'^2 = \frac{2}{3} a \Rightarrow x_2' = \sqrt{\frac{2}{3}} a \approx 0,81649a$$

$$w(x_2) = F_A^* \cdot x_2' - F^* \cdot \frac{x_2'}{3} = \frac{4}{9\sqrt{3}} \cdot \frac{F \cdot a^3}{EI_y} \approx 0,2566 \cdot \frac{F \cdot a^3}{EI_y}$$

(U skorijoj budućnosti, svi primjeri analognih greda bit će iscrtani i ispisani uobičajenom tehnikom, a sada se ovdje daju skenirani iz radnog materijala!).