

Primjer A: Deformacije ravnog nosača metodom analogne grede

Za ravni nosač ABCDE zadan i opterećen prema slici A) treba:

- a) odrediti reakcije u osloncima A i B nosača, te nacrtati dijagrame poprečnih sila i momenata savijanja duž nosača,
- b) odrediti metodom analogne grede progibe nosača i kutove nagiba tangente na elastičnu liniju nosača u svim označenim presjecima,
- c) skicirati i kotirati elastičnu liniju opterećenog nosača.

Zadano: F , a , $M = F \cdot a$, $EI_y = \text{konst.}$

Jedn. rovnice:

$$1. \sum \bar{F}_i = 0 \Rightarrow 2F + F_A + F_B = 0$$

$$2. \sum M_A = 0 \Rightarrow F \cdot a - \frac{q \cdot a^2}{2} - F \cdot 2a - F_B \cdot 3a = 0 \quad | :3a$$

$$F_B = \frac{F}{3} (1 - \frac{1}{2} - 2) = -\frac{F}{2}, \quad F_A = -2F - F_B = -\frac{3}{2}F$$

$M_A = -F \cdot a, \quad (M_{D,L}) = -F \cdot 2a + F_A \cdot a = -\frac{1}{2}F \cdot a, \quad M_C = 0$

$M_{D,D} = M_{D,L} + \frac{q \cdot a^2}{2} = -\frac{1}{2}F \cdot a + \frac{1}{2}F \cdot a = 0, \quad M_E = F_B \cdot a = \frac{1}{2}F \cdot a$

Opterećenje analogne grede:

$$F_1^* = \frac{1}{2} \cdot \frac{F \cdot a^2}{EI_y} = F_3^*, \quad F_2^* = F_4^* = F_5^* = \frac{1}{4} \cdot \frac{F \cdot a^2}{EI_y}$$

Reakcije analogne grede:

$$\overline{AB}^*: 1. \sum \bar{F}_i^* = 0 \Rightarrow F_A^* + F_B^* - F_2^* - F_3^* + F_4^* + F_5^* = 0$$

$$2. \sum M_A^* = 0 \Rightarrow F_B^* \cdot \frac{a}{3} + F_3^* \cdot \frac{a}{2} - 2F_4^* \cdot 2a - F_5^* \cdot 3a = 0 \quad | :3a$$

$$F_B^* = \frac{F \cdot a^2}{3EI_y} \left(\frac{1}{4} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{2} - 2 \cdot \frac{1}{4} \cdot 2 \right) = -\frac{2}{9} \cdot \frac{F \cdot a^2}{EI_y}$$

$$F_A^* = F_2^* - F_B^* = \frac{F \cdot a^2}{EI_y} \left(\frac{1}{4} + \frac{2}{9} \right) = \frac{17}{36} \cdot \frac{F \cdot a^2}{EI_y}$$

$\overline{CA}^*: F_C^* = F_A^* + F_1^* = \frac{F \cdot a^2}{EI_y} \left(\frac{17}{36} + \frac{1}{2} \right) = \frac{35}{36} \cdot \frac{F \cdot a^2}{EI_y}$

$$M_C^* = F_1^* \cdot \frac{2}{3}a + F_A^* \cdot a = \frac{F \cdot a^3}{EI_y} \left(\frac{1}{2} \cdot \frac{2}{3} + \frac{17}{36} \cdot 1 \right) = \frac{29}{36} \cdot \frac{F \cdot a^3}{EI_y}$$

Nagibi tangente na elastičnu liniju: $\alpha_i = -Q_i^*$

$$\alpha_A = -Q_A^* = F_A^* = \frac{17}{36} \cdot \frac{F \cdot a^2}{EI_y} \quad (0,47222), \quad \alpha_B = -Q_B^* = F_B^* = -\frac{2}{9} \cdot \frac{F \cdot a^2}{EI_y} \quad (0,22222), \quad \alpha_C = -Q_C^* = F_C^* = \frac{35}{36} \cdot \frac{F \cdot a^2}{EI_y} \quad (0,97222)$$

$$\alpha_D = -Q_D^* = F_A^* - F_2^* - F_3^* = \frac{F \cdot a^2}{EI_y} \left(\frac{17}{36} - \frac{1}{4} - \frac{1}{2} \right) = -\frac{10}{36} \cdot \frac{F \cdot a^2}{EI_y} = -0,27777 \quad (x_{m1} = 1,567a \rightarrow \eta(x_{m1}) = 0,12234)$$

$$\alpha_E = -Q_E^* = F_B^* - F_5^* = \frac{F \cdot a^2}{EI_y} \left(\frac{2}{9} - \frac{1}{4} \right) = -\frac{1}{36} \cdot \frac{F \cdot a^2}{EI_y} = -0,02777 \quad (x_{m2} = 3,058a \rightarrow \eta(x_{m2}) = 0,138675)$$

Progibi grede: $\eta_C = M_C^* = \frac{29}{36} \cdot \frac{F \cdot a^3}{EI_y}$, $\eta_A = \eta_B = 0$, $\eta_E = M_E^* = F_B^* \cdot a - F_5^* \cdot \frac{a}{3} = \frac{F \cdot a^3}{EI_y} \left(\frac{2}{9} - \frac{1}{4} \cdot \frac{1}{3} \right) = \frac{5}{36} \cdot \frac{F \cdot a^3}{EI_y}$

$$\eta_D = M_D^* = F_B^* \cdot 2a - 2F_4^* \cdot a = \frac{F \cdot a^3}{EI_y} \left(\frac{2}{9} \cdot 2 - 2 \cdot \frac{1}{4} \cdot 1 \right) = -\frac{2}{36} \cdot \frac{F \cdot a^3}{EI_y} = -0,05555$$

(U skorijoj budućnosti, svi primjeri analognih greda bit će iscrtani i ispisani uobičajenom tehnikom, a sada se ovdje daju skenirani iz radnog materijala!).