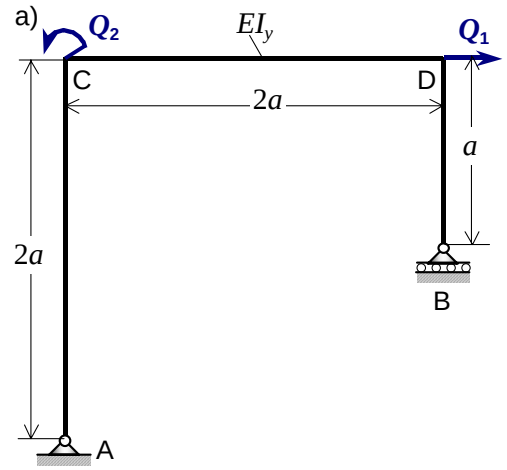
7. Primjer: Izračunavanje uplivnih koeficijenata za ravninski okvirni nosač



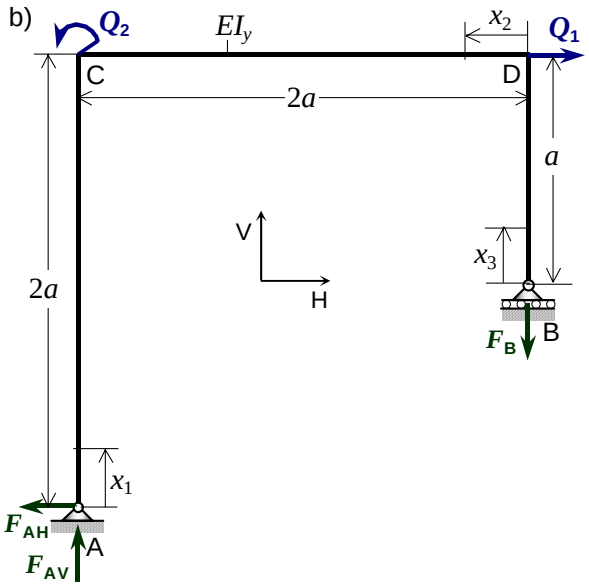
Za ravninski okvirni nosač ABCD zadan na slici a) treba odrediti uplivne koeficijente α_{ij}, za $i, j = 1,2$, uporabom drugog Castiglianovog poučka.

Zadano: a, EI_y=konst.

Rješenje:

Određivanje uplivnih koeficijenata uporabom drugog Castiglianovog poučka, kod konzolnog okvirnog nosača čiji dijelovi l_k, $k = 1,2,3$ imaju $EI_y = \text{konst.}$, slijedi iz izraza:

$$\alpha_{ij} = \left(\frac{\partial U}{\partial Q_i}\right)_{Q_j=1} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{Q_j=1} \cdot \frac{\partial M_y(x_k)}{\partial Q_i}\,\mathrm{d}x_k\right)\right].$$



Za određivanje momenata savijanja duž konture okvirnog nosača, potrebno je odrediti reakcije u osloncima A i B okvirnog nosača kod opterećenja poopćenim silama Q_1 i Q_2, slika b).

Reakcije u osloncima okvirnog nosača jesu:

1. $\sum F_{\mathrm{H}} = 0 \quad -F_{\mathrm{AH}} + Q_1 = 0 \rightarrow \quad F_{\mathrm{AH}} = Q_1$,
2. $\sum F_{\mathrm{V}} = 0 \quad F_{\mathrm{AV}} - F_{\mathrm{B}} = 0$, $\rightarrow \quad F_{\mathrm{AV}} = F_{\mathrm{B}}$,
3. $\sum M_{\mathrm{A}} = 0 \quad -F_{\mathrm{B}} \cdot 2a - Q_1 \cdot 2a + Q_2 = 0 \,/ : 2a$

Slijedi: $F_{\mathrm{B}} = -Q_1 + \dfrac{Q_2}{2a}$.

Momenti savijanja duž konture nosača $M_{\mathrm{b}}(x_k) = M_y(x_k)$ i potrebne derivacije jesu:

Momenti savijanja $M_y(x_k)$:	$\dfrac{\partial M_y(x_k)}{\partial Q_1}$	$\dfrac{\partial M_y(x_k)}{\partial Q_2}$
$M_y(x_1) = F_{\mathrm{AH}} \cdot x_1 = Q_1 \cdot x_1$	x_1	0
$M_y(x_2) = -F_{\mathrm{B}} \cdot x_2 = Q_1 \cdot x_2 - \dfrac{x_2}{2a} \cdot Q_2$	x_2	$-\dfrac{x_2}{2a}$
$M_y(x_3) = 0$	0	0

Uplivni koeficijenti nosača uporabom drugog Castiglianovog poučka jesu:

$$\alpha_{11} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=1\\Q_2=0}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=1\\Q_2=0}} \cdot \frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] =$$

$$= \frac{1}{EI_y}\left[\int_0^{2a} x_1 \cdot x_1 \,\mathrm{d}x_1 + \int_0^{2a} x_2 \cdot x_2 \,\mathrm{d}x_2\right] = \frac{16}{3}\cdot\frac{a^3}{EI_y},$$

$$\alpha_{12} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=1}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=1}} \cdot \frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\int_0^{2a} -\frac{1}{2a}x_2 \cdot x_2 \,\mathrm{d}x_2 = -\frac{4}{3}\cdot\frac{a^2}{EI_y} = \alpha_{21},$$

$$\alpha_{22} = \left(\frac{\partial U}{\partial Q_2}\right)_{\substack{Q_1=0\\Q_2=1}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=1}} \cdot \frac{\partial M_y(x_k)}{\partial Q_2}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\left[\int_0^{a} -\frac{1}{2a}x_2 \cdot \left(-\frac{x_2}{2a}\right)\mathrm{d}x_2\right] = \frac{2}{3}\cdot\frac{a}{EI_y}.$$

Numerički primjer: $Q_1 = 3\ \mathrm{kN}$, $Q_2 = 25\ \mathrm{kN{\cdot}m}$, $a = 2{,}5\ \mathrm{m}$, $E = 200\ \mathrm{GPa}$,
presjek nosača: I NP140, $(I_y = 573\ \mathrm{cm}^4)$.

Krutost na savijanje nosača jest:

$$EI_y = 2\times10^{11}\cdot 573\times10^{-8} = 1{,}146\times10^{6}\ \mathrm{N{\cdot}m^2}.$$

Uplivni koeficijenti zadanog okvirnog nosača su:

$$\alpha_{11} = \frac{16}{3}\cdot\frac{a^3}{EI_y} = \frac{16\cdot 2{,}5^3}{3\cdot 1{,}146}\times10^{-6} = 72{,}7167\times10^{-6}\ \mathrm{m/N},$$

$$\alpha_{21} = -\frac{4}{3}\cdot\frac{a^2}{EI_y} = -\frac{4\cdot 2{,}5^2}{3\cdot 1{,}146}\times10^{-6} = -7{,}27167\times10^{-6}\ \mathrm{1/N},$$

$$\alpha_{22} = \frac{2}{3}\cdot\frac{a}{EI_y} = \frac{2\cdot 2{,}5}{3\cdot 1{,}146}\times10^{-6} = 1{,}454334\times10^{-6}\ \mathrm{1/(N{\cdot}m)}.$$

Deformacije okvirnog nosača u točkama C i D određene pomoću uplivnih koeficijenata jesu:

$$q_1 = u_{\mathrm{D}} = \alpha_{11}\cdot Q_1 + \alpha_{12}\cdot Q_2 = 10^{-3}\left(72{,}7167\cdot 3 - 7{,}27167\cdot 25\right) = 36{,}358\times10^{-3}\ \mathrm{m} \cong 36{,}36\ \mathrm{mm},$$

$$q_2 = \alpha_{\mathrm{C}} = \alpha_{21}\cdot Q_1 + \alpha_{22}\cdot Q_2 = 10^{-3}\left(-7{,}27167\cdot 3 + 1{,}454334\cdot 25\right) = 14{,}54334\times10^{-3}\ \mathrm{rad} \cong 0{,}83^{\mathrm{o}}.$$