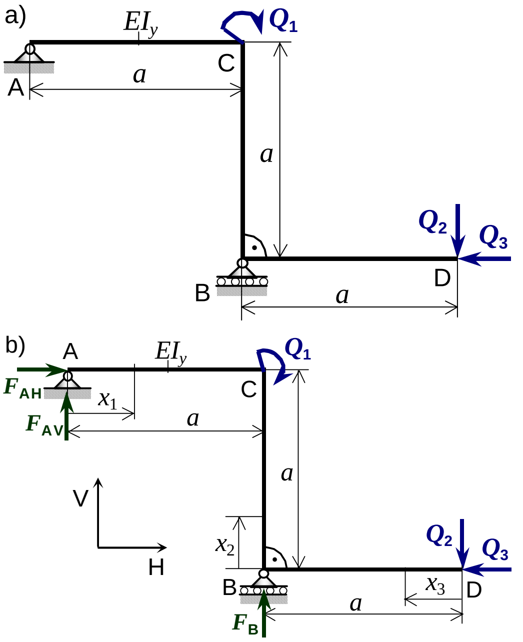
6. Primjer: Izračunavanje uplivnih koeficijenata za ravninski okvirni nosač



Za ravninski okvirni nosač ABCD zadan na slici a) treba odrediti uplivne koeficijente α_{ij}, za $i, j = 1,2,3$, uporabom drugog Castiglianovog poučka.

Zadano: a, EI_y=konst.

Rješenje:

Određivanje uplivnih koeficijenata uporabom drugog Castiglianovog poučka, kod konzolnog okvirnog nosača čiji dijelovi l_k, $k = 1,2,3$ imaju $EI_y = \text{konst.}$, slijedi iz izraza:

$$\alpha_{ij} = \left(\frac{\partial U}{\partial Q_i}\right)_{Q_j=1} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{Q_j=1}\cdot\frac{\partial M_y(x_k)}{\partial Q_i}\,\mathrm{d}x_k\right)\right].$$

Za određivanje momenata savijanja duž konture okvirnog nosača, potrebno je znati reakcije u osloncima A i B nosača kod opterećenja poopćenim silama Q_1, Q_2, Q_3, slika b). Reakcije u osloncima okvirnog nosača jesu:

$$\sum F_{\mathrm{H}} = 0 \quad F_{\mathrm{AH}} - Q_3 = 0 \;\rightarrow\; F_{\mathrm{AH}} = Q_3,$$

$$\sum F_{\mathrm{V}} = 0 \quad F_{\mathrm{AV}} + F_{\mathrm{B}} - Q_2 = 0,$$

$$\sum M_{\mathrm{A}} = 0 \quad F_{\mathrm{B}}\cdot a - Q_1 - Q_2\cdot 2a - Q_3\cdot a = 0 \,/ : a$$

Slijedi: $F_{\mathrm{B}} = \dfrac{Q_1}{a} + 2Q_2 + Q_3$, $\quad F_{\mathrm{A}} = Q_2 - F_{\mathrm{B}} = -\dfrac{Q_1}{a} - Q_2 - Q_3$.

Momenti savijanja duž konture nosača $M_{\mathrm{b}}(x_k) = M_y(x_k)$ i potrebne derivacije jesu:

Momenti savijanja $M_y(x_k)$:	$\frac{\partial M_y(x_k)}{\partial Q_1}$	$\frac{\partial M_y(x_k)}{\partial Q_2}$	$\frac{\partial M_y(x_k)}{\partial Q_3}$
$M_y(x_1) = F_{\mathrm{AV}}\cdot x_1 = -\frac{x_1}{a}Q_1 - Q_2\cdot x_1 - Q_3\cdot x_1$	$-\frac{x_1}{a}$	$-x_1$	$-x_1$
$M_y(x_2) = -Q_2\cdot a - Q_3\cdot x_2$	0	$-a$	$-x_2$
$M_y(x_3) = -Q_2\cdot x_3$	0	$-x_3$	0

Uplivni koeficijenti nosača uporabom drugog Castiglianovog poučka jesu:

$$\alpha_{11} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=1\\Q_2=0\\Q_3=0}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=1\\Q_2=0\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\int_0^a -\frac{x_1}{a}\cdot(-\frac{x_1}{a})\,\mathrm{d}x_1 = \frac{a}{3EI_y},$$

$$\alpha_{12} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\int_0^a -x_1\cdot(-\frac{x_1}{a})\,\mathrm{d}x_1 = \frac{a^2}{3EI_y} = \alpha_{21},$$

$$\alpha_{13}=\left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}=\frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right]=\frac{1}{EI_y}\int_0^a-\frac{x_1}{a}\cdot(-x_1)\,\mathrm{d}x_1=\frac{a^2}{3EI_y}=\alpha_{31}\,,$$

$$\alpha_{22}=\left(\frac{\partial U}{\partial Q_2}\right)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}=\frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_2}\,\mathrm{d}x_k\right)\right]=$$

$$=\frac{1}{EI_y}\left[\int_0^a -x_1\cdot(-x_1)\,\mathrm{d}x_1+\int_0^a -a\cdot(-a)\,\mathrm{d}x_2+\int_0^a -x_3\cdot(-x_3)\,\mathrm{d}x_3\right]=\frac{5}{3}\cdot\frac{a^3}{EI_y}\,,$$

$$\alpha_{23}=\left(\frac{\partial U}{\partial Q_2}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}=\frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_2}\,\mathrm{d}x_k\right)\right]=$$

$$=\frac{1}{EI_y}\left[\int_0^a -x_1\cdot(-x_1)\,\mathrm{d}x_1+\int_0^a -x_2\cdot(-a)\,\mathrm{d}x_2\right]=\frac{5}{6}\cdot\frac{a^3}{EI_y}=\alpha_{32}\,,$$

$$\alpha_{33}=\left(\frac{\partial U}{\partial Q_3}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}=\frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_3}\,\mathrm{d}x_k\right)\right]=$$

$$=\frac{1}{EI_y}\left[\int_0^a -x_1\cdot(-x_1)\,\mathrm{d}x_1+\int_0^a -x_2\cdot(-x_2)\,\mathrm{d}x_2\right]=\frac{2}{3}\cdot\frac{a^3}{EI_y}\,.$$