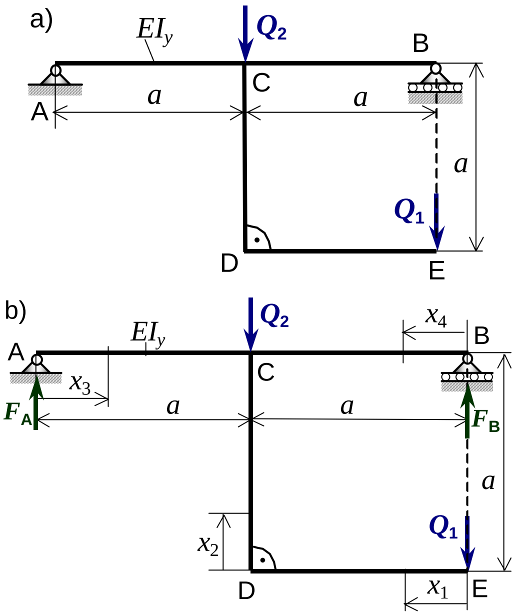
5. Primjer: Izračunavanje uplivnih koeficijenata za ravninski okvirni nosač



Za ravninski okvirni nosač ABCDE zadan na slici a) treba odrediti uplivne koeficijente α_{ij}, za $i, j = 1, 2$, uporabom drugog Castiglianovog poučka.

Zadano: a, EI_y=konst.

Rješenje:

Određivanje uplivnih koeficijenata uporabom drugog Castiglianovog poučka, kod konzolnog okvirnog nosača čiji dijelovi l_k, $k = 1,\ldots 4$ imaju $EI_y = \text{konst.}$, slijedi iz izraza:

$$\alpha_{ij} = \left(\frac{\partial U}{\partial Q_i}\right)_{Q_j=1} = \frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k} M_y(x_k)_{Q_j=1}\cdot\frac{\partial M_y(x_k)}{\partial Q_i}\,\mathrm{d}x_k\right)\right].$$

Za određivanje momenata savijanja duž konture okvirnog nosača, potrebno je znati reakcije u osloncima nosača kod opterećenja poopćenim silama Q_1 i Q_2, slika b). Reakcije u osloncima okvirnog nosača jesu:

$$\sum F_V = 0 \quad F_A + F_B - Q_1 - Q_2 = 0,$$

$$\sum M_B = 0 \quad -F_A\cdot 2a + Q_2\cdot a = 0 \,/ : 2a$$

Slijedi: $F_A = \dfrac{Q_2}{2}$, $F_B = Q_1 + \dfrac{Q_2}{2}$.

Momenti savijanja duž konture nosača $M_b(x_k) = M_y(x_k)$ i potrebne derivacije jesu:

Momenti savijanja $M_y(x_k)$:	$\dfrac{\partial M_y(x_k)}{\partial Q_1}$	$\dfrac{\partial M_y(x_k)}{\partial Q_2}$
$M_y(x_1) = -Q_1\cdot x_1$	$-x_1$	0
$M_y(x_2) = -Q_1\cdot a$	$-a$	0
$M_y(x_3) = F_A\cdot x_3 = \dfrac{Q_2}{2}\cdot x_3$	0	$\dfrac{x_3}{2}$
$M_y(x_4) = F_B\cdot x_4 = Q_1\cdot x_4 + \dfrac{Q_2}{2}\cdot x_4$	x_4	$\dfrac{x_4}{2}$

Uplivni koeficijenti nosača uporabom drugog Castiglianovog poučka jesu:

$$\alpha_{11} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=1\\Q_2=0}} = \frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=1\\Q_2=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] =$$

$$= \frac{1}{EI_y}\left[\int_0^a -x_1\cdot(-x_1)\,\mathrm{d}x_1 + \int_0^a -a\cdot(-a)\,\mathrm{d}x_2 + \int_0^a x_4\cdot x_4\,\mathrm{d}x_4\right] = \frac{5}{3}\cdot\frac{a^3}{EI_y},$$

$$\alpha_{12} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=1}} = \frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\int_0^a \frac{x_4}{2}\cdot x_4\,\mathrm{d}x_4 = \frac{a^3}{6EI_y} = \alpha_{21},$$

$$\alpha_{22} = \left(\frac{\partial U}{\partial Q_2} \right)_{\substack{Q_1=0 \\ Q_2=1}} = \frac{1}{EI_y} \left[\sum_{k=1}^{4} \left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0 \\ Q_2=1}} \cdot \frac{\partial M_y(x_k)}{\partial Q_2} \mathrm{d}x_k \right) \right] =$$

$$= \frac{1}{EI_y} \left[\int_0^a \frac{x_3}{2} \cdot \frac{x_3}{2} \mathrm{d}x_3 + \int_0^a \frac{x_4}{2} \cdot \frac{x_4}{2} \mathrm{d}x_4 \right] = \frac{a^3}{6EI_y} .$$