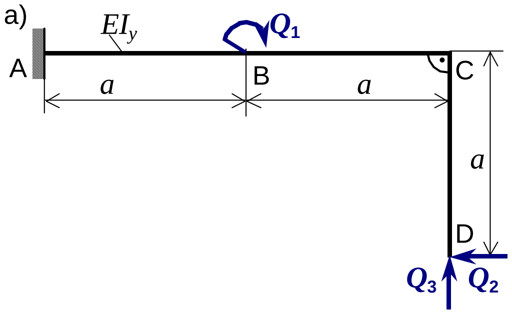
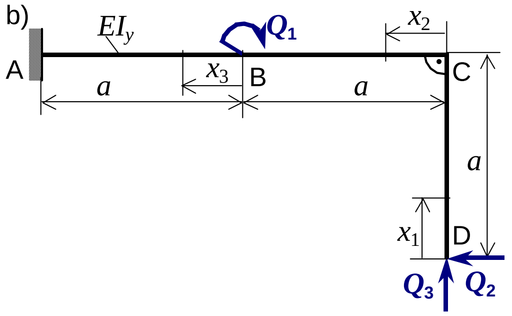
## 4. Primjer: Izračunavanje uplivnih koeficijenata za ravninski konzolni okvirni nosač




Za ravninski konzolni okvirni nosač ABCD zadan na slici a) treba odrediti uplivne koeficijente $\alpha_{ij}$, za $i, j = 1, 2, 3$, uporabom drugog Castiglianovog poučka.

Zadano: $a$, $EI_y$=konst.

**Rješenje:**




Određivanje uplivnih koeficijenata uporabom drugog Castiglianovog poučka, kod konzolnog okvirnog nosača čiji dijelovi $l_k$, $k = 1, 2, 3$ imaju $EI_y = \text{konst.}$, slijedi iz izraza:

$$\alpha_{ij} = \left(\frac{\partial U}{\partial Q_i}\right)_{Q_j=1} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{Q_j=1}\cdot\frac{\partial M_y(x_k)}{\partial Q_i}\,\mathrm{d}x_k\right)\right].$$

Momenti savijanja duž konture nosača $M_b(x_k) = M_y(x_k)$ i potrebne derivacije jesu:

| Momenti savijanja $M_y(x_k)$: | $\dfrac{\partial M_y(x_k)}{\partial Q_1}$ | $\dfrac{\partial M_y(x_k)}{\partial Q_2}$ | $\dfrac{\partial M_y(x_k)}{\partial Q_3}$ |
|---|---|---|---|
| $M_y(x_1) = -Q_2 \cdot x_1$ | $0$ | $-x_1$ | $0$ |
| $M_y(x_2) = -Q_2 \cdot a + Q_3 \cdot x_2$ | $0$ | $-a$ | $x_2$ |
| $M_y(x_3) = -Q_1 - Q_2 \cdot a + Q_3 \cdot (a + x_3)$ | $-1$ | $-a$ | $(a + x_3)$ |

Uplivni koeficijenti nosača uporabom drugog Castiglianovog poučka jesu:

$$\alpha_{11} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=1\\Q_2=0\\Q_3=0}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=1\\Q_2=0\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\int_0^a -1\cdot(-1)\,\mathrm{d}x_3 = \frac{a}{EI_y},$$

$$\alpha_{12} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\int_0^a -a\cdot(-1)\,\mathrm{d}x_3 = \frac{a^2}{EI_y} = \alpha_{21},$$

$$\alpha_{13} = \left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\,\mathrm{d}x_k\right)\right] = \frac{1}{EI_y}\int_0^a (a+x_3)\cdot(-1)\,\mathrm{d}x_3 = -\frac{3}{2}\cdot\frac{a^2}{EI_y} = \alpha_{31},$$

$$\alpha_{22} = \left(\frac{\partial U}{\partial Q_2}\right)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_2}\,\mathrm{d}x_k\right)\right] =$$

$$= \frac{1}{EI_y}\left[\int_0^a -x_1\cdot(-x_1)\,\mathrm{d}x_1 + \int_0^a -a\cdot(-a)\,\mathrm{d}x_2 + \int_0^a -a\cdot(-a)\,\mathrm{d}x_3\right] = \frac{7}{3}\cdot\frac{a^3}{EI_y},$$

$$\alpha_{23} = \left(\frac{\partial U}{\partial Q_2}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}} \cdot \frac{\partial M_y(x_k)}{\partial Q_2}\,\mathrm{d}x_k\right)\right] =$$

$$= \frac{1}{EI_y}\left[\int_0^a x_2 \cdot (-a)\,\mathrm{d}x_2 + \int_0^a (a+x_3)\cdot(-a)\,\mathrm{d}x_3\right] = -2\cdot\frac{a^3}{EI_y} = \alpha_{32}\,,$$

$$\alpha_{33} = \left(\frac{\partial U}{\partial Q_3}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}} = \frac{1}{EI_y}\left[\sum_{k=1}^{3}\left(\int_0^{l_k} M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}} \cdot \frac{\partial M_y(x_k)}{\partial Q_3}\,\mathrm{d}x_k\right)\right] =$$

$$= \frac{1}{EI_y}\left[\int_0^a x_2 \cdot x_2\,\mathrm{d}x_2 + \int_0^a (a+x_3)\cdot(a+x_3)\,\mathrm{d}x_3\right] = \frac{8}{3}\cdot\frac{a^3}{EI_y}\,.$$