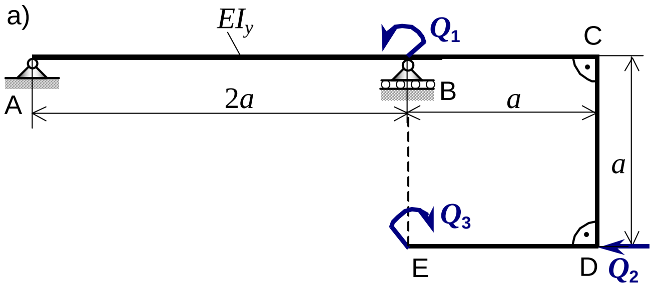
3. Primjer: Izračunavanje uplivnih koeficijenata za ravninski okvirni nosač

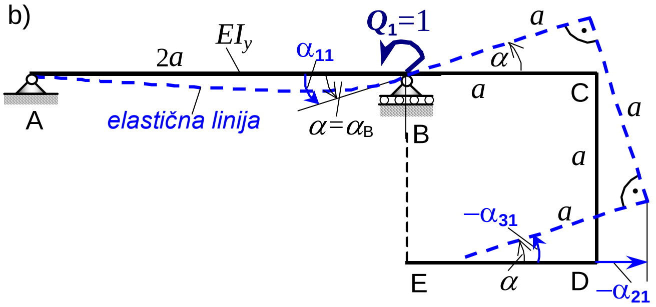


Za ravninski okvirni nosač ABCDE zadan na slici a) treba odrediti:

1. uplivne koeficijente α_{11}, α_{12} i α_{13} primjenom metode deformacija,
2. uplivne koeficijente α_{ij}, $i, j = 1, 2, 3$, uporabom drugog Castiglianovog poučka.

Zadano: a, EI_y=konst.

Rješenje:



1. Uplivne koeficijenti α_{11}, α_{12} i α_{13} može se odrediti pomoću tabličnih izraza u literaturi ili metodom analogne grede za vrijednost poopćene sile $Q_1 = 1$, slika b).

Na slici b) prikazana je elastična linija okvirnog nosača kod opterećenja nosača poopćenom silom $Q_1 = 1$, te deformacije na mjestima poopćenih sila Q_2 i Q_3.

Kut nagiba α nosača AB na mjestu oslonca B kod opterećenja jediničnim momentom na tom osloncu, slika b), pomoću tabličnog izraza jest ($l = 2a$):

$$\alpha = \frac{1}{3} \cdot \frac{\mathcal{M} \cdot l}{EI_y} = \frac{1}{3} \cdot \frac{Q_1 l}{EI_y} = \frac{1}{3} \cdot \frac{1 \cdot 2a}{EI_y} = \frac{2}{3} \cdot \frac{a}{EI_y} .$$

Uplivni su koeficijenti za opterećenje nosača poopćenom silom $Q_1 = 1$, slika b):

$$\alpha_{11} = \alpha = \frac{2}{3} \cdot \frac{a}{EI_y} .$$

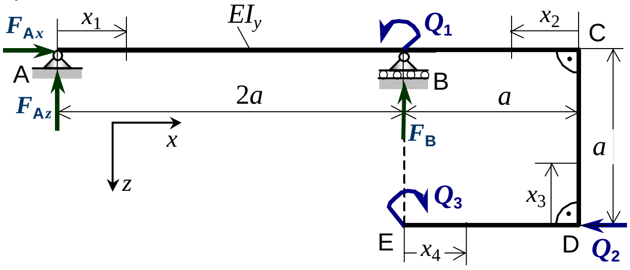
Preostali uplivni koeficijenti mogu se odrediti metodom deformacija, slika b):

$$\alpha_{12} = \alpha_{21} = -a \cdot \alpha = -\frac{2}{3} \cdot \frac{a^2}{EI_y}, \quad \alpha_{13} = \alpha_{31} = -\alpha = -\frac{2}{3} \cdot \frac{a}{EI_y} .$$

2. Određivanje uplivnih koeficijenata uporabom drugog Castiglianovog poučka, kod okvirnog nosača čiji dijelovi l_k, $k = 1,\ldots 4$ imaju $EI_y = \text{konst.}$, slijedi iz izraza:

$$\alpha_{ij} = \left(\frac{\partial U}{\partial Q_i} \right)_{Q_j = 1} = \frac{1}{EI_y} \left[\sum_{k=1}^{4} \left(\int_0^{l_k} M_y(x_k)_{Q_j=1} \cdot \frac{\partial M_y(x_k)}{\partial Q_i} \mathrm{d}x_k \right) \right].$$

Za određivanje momenata savijanja duž konture okvirnog nosača, potrebno je znati reakcije u osloncima nosača kod opterećenja poopćenim silama Q_1, Q_2 i Q_3, slika c).



Reakcije u osloncima nosača jesu:

$F_{Ax} = Q_2$,

$\sum M_B = 0 \;\; -F_{Az} \cdot 2a + Q_1 - Q_2 \cdot a - Q_3 = 0 \, / : 2a$

Slijedi: $F_{Az} = \frac{Q_1}{2a} - \frac{Q_2}{2} - \frac{Q_3}{2a} = F_B$.

Momenti savijanja duž konture nosača $M_{\mathrm{b}}(x_k)=M_y(x_k)$ i potrebne derivacije jesu:

Momenti savijanja $M_y(x_k)$:	$\dfrac{\partial M_y(x_k)}{\partial Q_1}$	$\dfrac{\partial M_y(x_k)}{\partial Q_2}$	$\dfrac{\partial M_y(x_k)}{\partial Q_3}$
$M_y(x_1)=F_{\mathrm{Az}}\cdot x_1=\dfrac{Q_1}{2a}\cdot x_1-\dfrac{Q_2}{2}\cdot x_1-\dfrac{Q_3}{2a}\cdot x_1$	$\dfrac{x_1}{2a}$	$-\dfrac{x_1}{2}$	$-\dfrac{x_1}{2a}$
$M_y(x_2)=-Q_2\cdot a-Q_3$	0	$-a$	-1
$M_y(x_3)=-Q_2\cdot x_3-Q_3$	0	$-x_3$	-1
$M_y(x_4)=-Q_3$	0	0	-1

Uplivni koeficijenti nosača uporabom drugog Castiglianovog poučka jesu:

$$\alpha_{11}=\left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=1\\Q_2=0\\Q_3=0}}=\frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k}M_y(x_k)_{\substack{Q_1=1\\Q_2=0\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\mathrm{d}x_k\right)\right]=\frac{1}{EI_y}\int_0^{2a}\frac{1}{2a}\cdot x_1\cdot\frac{x_1}{2a}\mathrm{d}x_1=\frac{2}{3}\cdot\frac{a}{EI_y},$$

$$\alpha_{12}=\left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}=\frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k}M_y(x_k)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\mathrm{d}x_k\right)\right]=\frac{1}{EI_y}\int_0^{2a}-\frac{x_1}{2}\cdot\frac{x_1}{2a}\mathrm{d}x_1=-\frac{2}{3}\cdot\frac{a^2}{EI_y}=\alpha_{21},$$

$$\alpha_{13}=\left(\frac{\partial U}{\partial Q_1}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}=\frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k}M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_1}\mathrm{d}x_k\right)\right]=\frac{1}{EI_y}\int_0^{2a}-\frac{1}{2a}\cdot x_1\cdot\frac{x_1}{2a}\mathrm{d}x_1=-\frac{2}{3}\cdot\frac{a}{EI_y}=\alpha_{31},$$

$$\alpha_{22}=\left(\frac{\partial U}{\partial Q_2}\right)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}=\frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k}M_y(x_k)_{\substack{Q_1=0\\Q_2=1\\Q_3=0}}\cdot\frac{\partial M_y(x_k)}{\partial Q_2}\mathrm{d}x_k\right)\right]=$$

$$=\frac{1}{EI_y}\left[\int_0^{2a}-\frac{1}{2}\cdot x_1\cdot\frac{-x_1}{2}\mathrm{d}x_1+\int_0^{a}-a\cdot(-a)\mathrm{d}x_2+\int_0^{a}-x_3\cdot(-x_3)\mathrm{d}x_3\right]=2\frac{a^3}{EI_y},$$

$$\alpha_{23}=\left(\frac{\partial U}{\partial Q_2}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}=\frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k}M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_2}\mathrm{d}x_k\right)\right]=$$

$$=\frac{1}{EI_y}\left[\int_0^{2a}-\frac{1}{2a}\cdot x_1\cdot\frac{-x_1}{2}\mathrm{d}x_1+\int_0^{a}-1\cdot(-a)\mathrm{d}x_2+\int_0^{a}-1\cdot(-x_3)\mathrm{d}x_3\right]=\frac{13}{6}\cdot\frac{a^2}{EI_y}=\alpha_{32},$$

$$\alpha_{33}=\left(\frac{\partial U}{\partial Q_3}\right)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}=\frac{1}{EI_y}\left[\sum_{k=1}^{4}\left(\int_0^{l_k}M_y(x_k)_{\substack{Q_1=0\\Q_2=0\\Q_3=1}}\cdot\frac{\partial M_y(x_k)}{\partial Q_3}\mathrm{d}x_k\right)\right]=$$

$$=\frac{1}{EI_y}\left[\int_0^{2a}-\frac{1}{2a}\cdot x_1\cdot\frac{-x_1}{2a}\mathrm{d}x_1+\int_0^{a}-1\cdot(-1)\mathrm{d}x_2+\int_0^{a}-1\cdot(-1)\mathrm{d}x_3+\int_0^{a}-1\cdot(-1)\mathrm{d}x_4\right]=\frac{11}{3}\cdot\frac{a}{EI_y}.$$