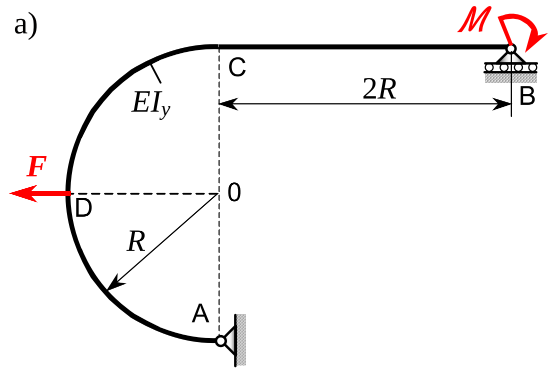
9. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

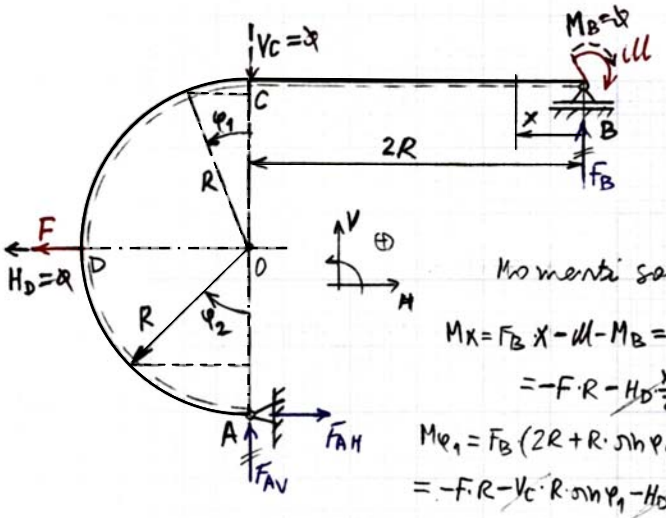


Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba:

- odrediti reakcije veza u osloncima A i B
- odrediti vertikalni pomak u C (w_C = ?)
- odrediti vodoravni pomak u D (u_D = ?)
- odrediti kutni zakret na mjestu oslonca B (α_B = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: $\boldsymbol{F}$, R, $\mathcal{M} = F \cdot R$, EI_y = konst.

Rješenje:



Jedn. ravnoteže:

1. $\Sigma F_H = 0 \quad F_{AH} - F - H_B = 0 \rightarrow F_{AH} = F + H_D \rightarrow F_{AH} = F$
2. $\Sigma F_V = 0 \quad F_{AV} + F_B - V_C = 0 \rightarrow F_{AV} = 0 + \frac{H_D}{2} + V_C - \frac{M_B}{2R}$
3. $\Sigma M_A = 0 \quad \cancel{F \cdot R} + H_D \cdot R - \cancel{\mathcal{M}} - M_B + F_B \cdot 2R = 0 \ / : 2R$

$$F_B = 0 - \frac{H_D}{2} + \frac{M_B}{2R} \rightarrow F_B = 0$$

Momenti savijanja za derivacije i pomake:

$$M_x = F_B \cdot x - \mathcal{M} - M_B = -\frac{H_D}{2} \cdot x + \frac{M_B}{2R} \cdot x - F \cdot R - M_B =$$
$$= -F \cdot R - \cancel{H_D} \frac{x}{2} - \cancel{M_B}\left(1 - \frac{x}{2R}\right)$$

$$M_{\varphi_1} = F_B\,(2R + R \cdot \sin\varphi_1) - V_C \cdot R \cdot \sin\varphi_1 - \mathcal{M} - M_B =$$
$$= -F \cdot R - \cancel{V_C} \cdot R \cdot \sin\varphi_1 - \cancel{H_D} R\left(1 + \frac{\sin\varphi_1}{2}\right) + \cancel{M_B} \cdot \frac{\sin\varphi_1}{2}$$

$$M_{\varphi_2} = -F_{AV} \cdot R \cdot \sin\varphi_2 - F_{AH} \cdot R\,(1 - \cos\varphi_2) = -F \cdot R\,(1 - \cos\varphi_2) - \cancel{V_C} \cdot R \cdot \sin\varphi_2 -$$
$$- \cancel{H_D} R\,(1 + \sin\varphi_2 - \cos\varphi_2) + \cancel{M_B} \cdot \frac{\sin\varphi_2}{2}$$

$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_D}$	$\frac{\partial M_i}{\partial M_B}$
0	$-\frac{x}{2}$	$-\left(1 - \frac{x}{2R}\right)$
$-R \cdot \sin\varphi_1$	$-R\left(1 + \frac{\sin\varphi_1}{2}\right)$	$\frac{\sin\varphi_1}{2}$
$-R \cdot \sin\varphi_2$	$-R\,(1 + \sin\varphi_2 - \cos\varphi_2)$	$\frac{\sin\varphi_2}{2}$

Vertikalni pomak u C:

$$w_C = \left(\frac{\partial U}{\partial V_C}\right)_{V_C=0} = \frac{F}{EI_y}\left[\int_0^{\frac{\pi}{2}} +R \cdot R \cdot \sin\varphi_1 \cdot R\,d\varphi_1 + \int_0^{\frac{\pi}{2}} R\,(1 - \cos\varphi_2) \cdot R \cdot \sin\varphi_2 \cdot R\,d\varphi_2\right) = \frac{FR^3}{EI_y}\left(1 + 1 - \frac{1}{2}\right) = \frac{3}{2} \cdot \frac{FR^3}{EI_y} \ (\downarrow)$$

Vodoravni pomak u D:

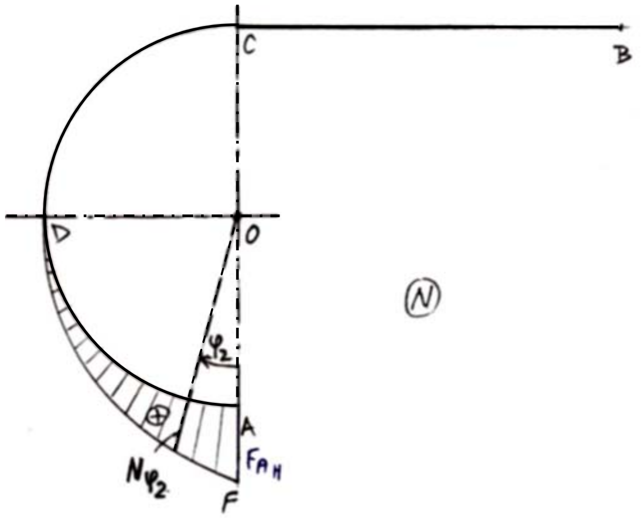
$$u_D = \left(\frac{\partial U}{\partial H_D}\right)_{H_D=0} = \frac{F}{EI_y}\left[\int_0^{2R} R \cdot \frac{x}{2}\,dx + \int_0^{\pi/2} R \cdot R\left(1 + \frac{1}{2} \cdot \sin\varphi_1\right) \cdot R\,d\varphi_1 + \int_0^{\frac{\pi}{2}} R\,(1 - \cos\varphi_2) \cdot R\,(1 + \sin\varphi_2 - \cos\varphi_2)\,R\,d\varphi_2\right] =$$
$$= \frac{FR^3}{EI_y}\left(\frac{1}{2} \cdot 2 + \frac{\pi}{2} + \frac{1}{2} \cdot 1 + \frac{\pi}{2} + 1 - 1 - 1 - \frac{1}{2} + \frac{\pi}{4}\right) = \frac{5\pi}{4}\frac{FR^3}{EI_y} \cong 3{,}927\,\frac{FR^3}{EI_y} \ (\leftarrow)$$

Kutni zakret u B:

$$\alpha_B = \left(\frac{\partial U}{\partial M_B}\right)_{M_B=0} = \frac{F}{EI_y}\left[\int_0^{2R} R \cdot \left(1 - \frac{x}{2R}\right)dx + \int_0^{\frac{\pi}{2}} -R \cdot \frac{\sin\varphi_1}{2} \cdot R\,d\varphi_1 + \int_0^{\frac{\pi}{2}} -R\,(1 - \cos\varphi_2) \cdot \frac{\sin\varphi_2}{2} \cdot R\,d\varphi_2\right] =$$
$$= \frac{FR^2}{EI_y}\left(2 - \frac{1}{2} \cdot 2 - \frac{1}{2} \cdot 1 - \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{1}{4} \cdot \frac{FR^2}{EI_y} \ (\circlearrowleft)$$

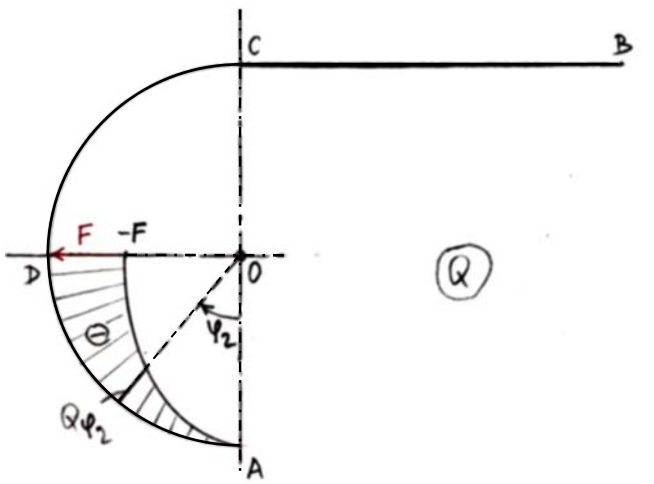
Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

Dijagrami unutarnjih sila duž konture okvirnog nosača:



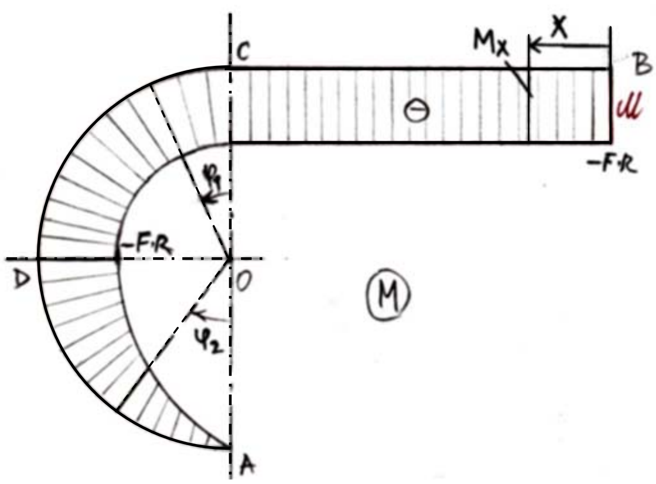
$N_x = N_{\varphi_1} = 0$

$N_{\varphi_2} = F \cdot \cos\varphi_2 \;, \quad 0 \le \varphi_2 \le \frac{\pi}{2}$



$Q_x = Q_{\varphi_1} = 0$

$Q_{\varphi_2} = -F \cdot \sin\varphi_2 \;, \quad 0 \le \varphi_2 \le \frac{\pi}{2}$



$M_x = -F \cdot R \;, \quad M_{\varphi_1} = -F \cdot R \;, \quad 0 \le \varphi_1 \le \frac{\pi}{2}$

$M_{\varphi_2} = -FR(1 - \cos\varphi_2) \;, \quad 0 \le \varphi_2 \le \frac{\pi}{2}$

$M_B = M_C = M_D = -F \cdot R \;, \quad M_A = 0$

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).