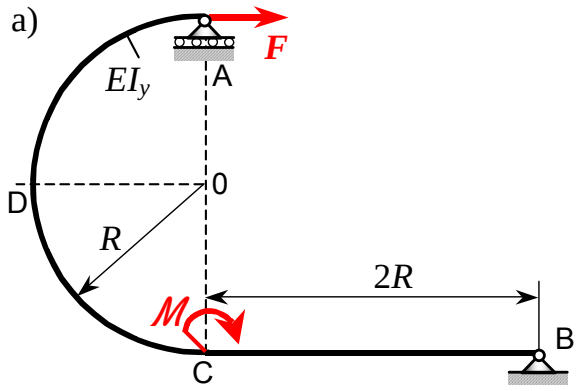
8. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

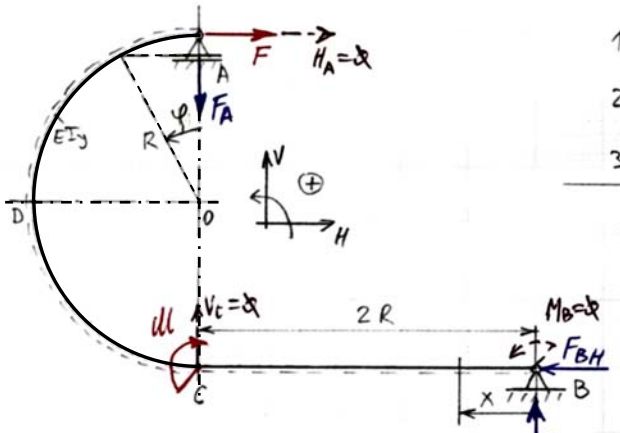


Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba odrediti:

- reakcije veza u osloncima A i B
- vertikalni pomak u C (w_C = ?)
- vodoravni pomak oslonca A (u_A = ?)
- odrediti kutni zakret na mjestu oslonca B (α_B = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: F, R, $M = F \cdot R$, EI_y = konst.

Rješenje:



Reakcije u osloncima:

1. $\Sigma F_H = 0 \quad F + H_A - F_{BH} = 0 \rightarrow F_{BH} = F + H_A$

2. $\Sigma F_V = 0 \quad -F_A + V_C + F_{BV} = 0$

3. $\Sigma M_B = 0 \quad F_A \cdot 2R - V_C \cdot 2R - M - F \cdot 2R - H_A \cdot 2R + M_B = 0 \;/ : 2R$

$$F_A = \frac{3}{2}F + V_C + H_A - \frac{M_B}{2R}, \quad F_{BV} = F_A - V_C = \frac{3}{2}F + H_A - \frac{M_B}{2R}$$

Momenti savijanja i derivacije:

$$M(x) = F_{BV} \cdot x + M_B = \frac{3}{2}F \cdot x + H_A \cdot x + M_B\left(1 - \frac{x}{2R}\right)$$

$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_A}$	$\frac{\partial M_i}{\partial M_B}$
0	x	$\left(1 - \frac{x}{2R}\right)$
$R\sin\varphi$	$R(1+\sin\varphi - \cos\varphi)$	$-\frac{\sin\varphi}{2}$

$$M(\varphi) = F_A \cdot R \cdot \sin\varphi + FR(1-\cos\varphi) + H_A R(1-\cos\varphi) = FR\left(1 + \frac{3}{2}\sin\varphi - \cos\varphi\right) + V_C R \sin\varphi + H_A R(1 + \sin\varphi - \cos\varphi) - M_B \frac{\sin\varphi}{2}$$

Vertikalni pomak u C:

$$w_C = \left(\frac{\partial U}{\partial V_C}\right)_{V_C=0} = \frac{FR^3}{EI_y}\int_0^{\pi}\left(1 + \frac{3}{2}\sin\varphi - \cos\varphi\right)\cdot\sin\varphi\, d\varphi = \frac{FR^3}{EI_y}\left(2 + \frac{3}{2}\cdot\frac{\pi}{2} - 0\right) = \left(2 + \frac{3\pi}{4}\right)\frac{FR^3}{EI_y} \approx 4{,}3562\,\frac{FR^3}{EI_y}\;(\downarrow)$$

Vodoravni pomak u A:

$$u_A = \left(\frac{\partial U}{\partial H_A}\right)_{H_A=0} = \frac{F}{EI_y}\left[\int_0^{2R}\frac{3}{2}x \cdot x\, dx + \int_0^{\pi} R\left(1 + \frac{3}{2}\sin\varphi - \cos\varphi\right)\cdot R\left(1 + \sin\varphi - \cos\varphi\right) R\, d\varphi\right] =$$

$$= \frac{FR^3}{EI_y}\left(\frac{3}{2}\cdot\frac{8}{3} + \pi + 2 - 0 + \frac{3}{2}\cdot 2 + \frac{3}{2}\cdot\frac{\pi}{2} - \frac{3}{2}\cdot 0 - 0 - 0 + \frac{\pi}{2}\right) = \left(3 + \frac{9\pi}{4}\right)\cdot\frac{FR^3}{EI_y} \approx 16{,}0686\,\frac{FR^3}{EI_y}\;(\rightarrow)$$

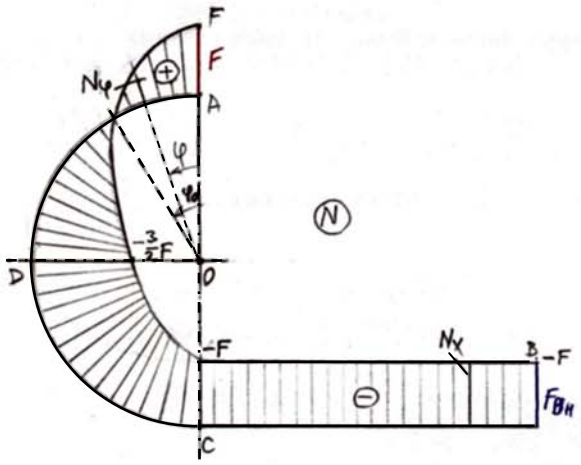
Kutni zakret u B:

$$\alpha_B = \left(\frac{\partial U}{\partial M_B}\right)_{M_B=0} = \frac{F}{EI_y}\left[\int_0^{2R}\frac{3}{2}x\cdot\left(1 - \frac{x}{2R}\right)dx + \int_0^{\pi} R\left(1 + \frac{3}{2}\sin\varphi - \cos\varphi\right)\cdot\left(-\frac{\sin\varphi}{2}\right)R\, d\varphi\right] = \frac{FR^2}{EI_y}\left(\frac{3}{2}\cdot 2 - \frac{3}{4}\cdot\frac{8}{3} - \frac{1}{2}\cdot 2 - \right.$$

$$\left. - \frac{3}{4}\cdot\frac{\pi}{2} + \frac{1}{2}\cdot 0\right) = -\frac{3}{8}\pi\cdot\frac{FR^2}{EI_y} \approx -1{,}178\cdot\frac{FR^2}{EI_y}\;(\circlearrowleft)$$

Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

Dijagrami unutarnjih sila duž konture okvirnog nosača:

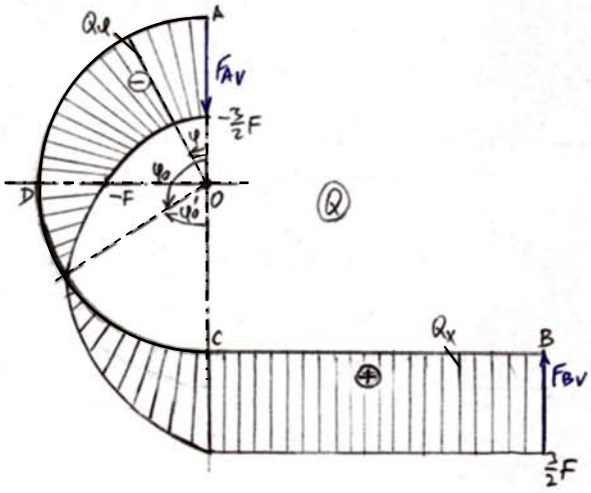


$N_X = -F_{BH} = -F$, $N_B = N_C = -F$

$N_\varphi = F \cdot \cos\varphi - \frac{3}{2}F \cdot \sin\varphi$, $0 \le \varphi \le \pi$

$N_A = F$, $N_D = -\frac{3}{2}F$

$N_\varphi = 0 \rightarrow \tan\varphi = \frac{2}{3} \rightarrow \varphi_0 \approx 33{,}7°$



$Q_X = F_{BV} = \frac{3F}{2}$, $0 \le X \le 2R$

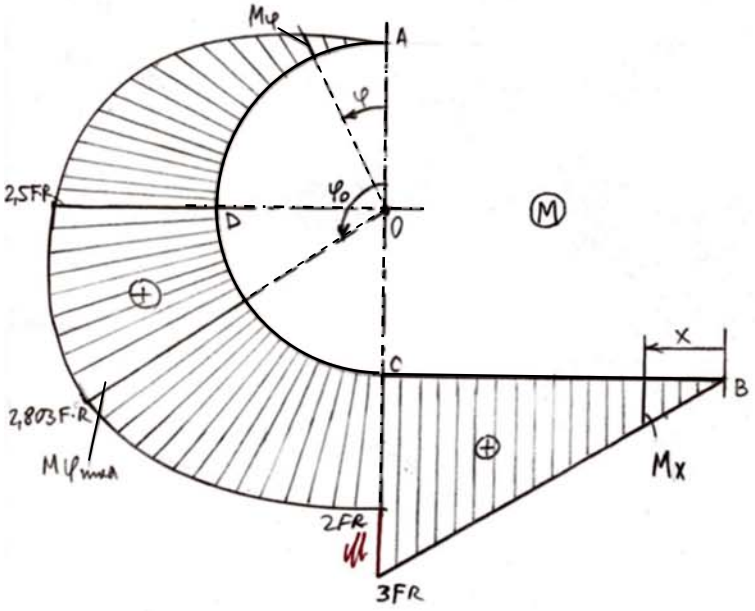
$Q_B = Q_C = \frac{3F}{2}$

$Q_\varphi = -F \cdot \sin\varphi - \frac{3}{2} \cdot F \cdot \cos\varphi$, $0 \le \varphi \le \pi$

$Q_A = -\frac{3}{2}F$, $Q_D = -F$

$Q_\varphi = 0 \rightarrow \tan\varphi_0 = -\frac{3}{2} \rightarrow \varphi_0 = 123{,}7°$

$\varphi_0' = 56{,}3°$



$M_X = F_{BV} \cdot X = \frac{3}{2}F \cdot X$, $X \le 2R$

$M_B = 0$, $M_{C,D} = 3FR$, $M_{C,L} = M_{C,D} - M_L = 2FR$,

$M_\varphi = F_{AV} \cdot R \cdot \sin\varphi + F \cdot R(1 - \cos\varphi) =$

$= FR\left(1 + \frac{3}{2}\sin\varphi - \cos\varphi\right)$, $0 \le \varphi \le \pi$

$M_D = 2{,}5F \cdot R$, $\varphi_0 = 123{,}7°$, $M_A = 0$

$M_{\varphi_0} \approx 2{,}803\ F \cdot R = M_{\varphi max}$

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).