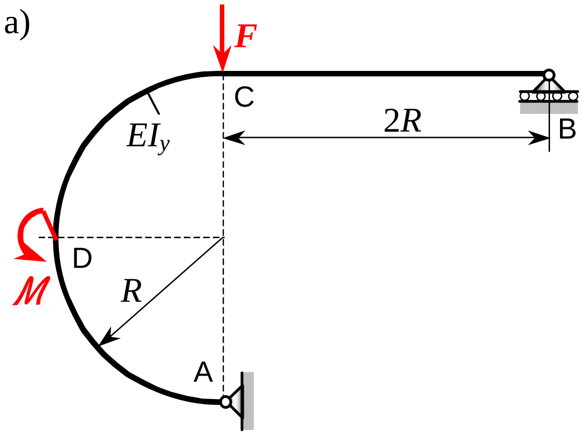
7. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

a)

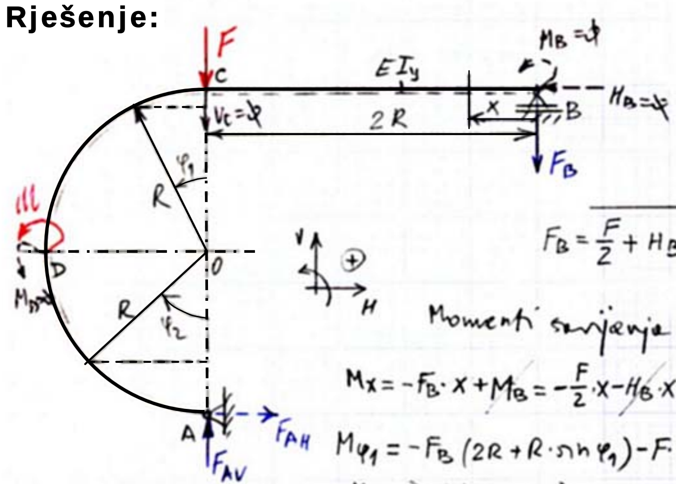


Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba odrediti:

- reakcije veza u osloncima A i B
- vertikalni pomak u C (w_C = ?)
- vodoravni pomak u točki B (u_B = ?)
- odrediti kutne zakrete u točkama B i D (α_B = ?, α_D = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: F, R, $\mathcal{M} = F\cdot R$, EI_y = konst.

Rješenje:



Jednadžbe ravnoteže:

1. $\Sigma F_H = 0 \quad F_{AH} = H_B = 0$
2. $\Sigma F_V = 0 \quad F_{AV} - F_B - F - V_C = 0$
3. $\Sigma M_A = 0 \quad -F_B \cdot 2R + \mathcal{M} + H_B \cdot 2R + M_B + M_D = 0 \;/ : 2R$

$$F_B = \frac{F}{2} + H_B + \frac{M_B}{2R} + \frac{M_D}{2R}\,, \quad F_{AV} = F + F_B + V_C = \frac{3}{2}F + V_C + H_B + \frac{M_B}{2R} + \frac{M_D}{2R}$$

Momenti savijanja i derivacije:

$$M_x = -F_B \cdot x + M_B = -\frac{F}{2}\cdot x - H_B\cdot x + M_B\left(1-\frac{x}{2R}\right) - M_D\cdot\frac{x}{2R}$$

$$M_{\varphi_1} = -F_B(2R + R\cdot\sin\varphi_1) - F\cdot R\cdot\sin\varphi_1 - V_C\cdot R\cdot\sin\varphi_1 + M_B + H_B R(1-\cos\varphi_1) = -FR\left(1+\frac{3}{2}\sin\varphi_1\right) - V_C\cdot R\cdot\sin\varphi_1 - H_B\cdot R(1+\sin\varphi_1+\cos\varphi_1) - \frac{M_B}{2}\sin\varphi_1 - \frac{M_D}{2}(2+\sin\varphi_1)$$

$$M_{\varphi_2} = -F_{AV}\cdot R\cdot\sin\varphi_2 - F_{AH} R(1-\cos\varphi_2) = -\frac{3}{2}FR\sin\varphi_2 - V_C\cdot R\cdot\sin\varphi_2 - H_B R(1+\sin\varphi_2-\cos\varphi_2) - \frac{M_B}{2}\cdot\sin\varphi_2 - \frac{M_D}{2}\cdot\sin\varphi_2$$

$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_B}$	$\frac{\partial M_i}{\partial M_B}$	$\frac{\partial M_i}{\partial M_D}$
0	$-x$	$\left(1-\frac{x}{2R}\right)$	$-\frac{x}{2R}$
$-R\cdot\sin\varphi_1$	$-R\cdot(1+\sin\varphi_1+\cos\varphi_1)$	$-\frac{\sin\varphi_1}{2}$	$-\left(1+\frac{\sin\varphi_1}{2}\right)$
$-R\cdot\sin\varphi_2$	$-R(1+\sin\varphi_2-\cos\varphi_2)$	$-\frac{\sin\varphi_2}{2}$	$-\frac{\sin\varphi_2}{2}$

Vertikalni pomak u C:

$$w_C = \left(\frac{\partial U}{\partial V_C}\right)_{V_C=0} = \frac{1}{EI_y}\left[\int_0^{\frac{\pi}{2}} -FR\left(1+\frac{3}{2}\sin\varphi_1\right)(-R\cdot\sin\varphi_1)R\,d\varphi_1 + \int_0^{\frac{\pi}{2}} -\frac{3}{2}FR\sin\varphi_2(-R\sin\varphi_2)R\,d\varphi_2\right] = \frac{FR^3}{EI_y}\left[1+\frac{3}{2}\cdot\frac{\pi}{4} + \frac{3}{2}\cdot\frac{\pi}{4}\right] = \frac{FR^3}{EI_y}\left(1+\frac{3\pi}{4}\right) \approx 3{,}3562\cdot\frac{FR^3}{EI_y}\;(\downarrow)$$

Vodoravni pomak oslonca B:

$$u_B = \left(\frac{\partial U}{\partial H_B}\right)_{H_B=0} = \frac{1}{EI_y}\left[\int_0^{2R} -\frac{F}{2}x\cdot(-x)\,dx + \int_0^{\pi/2} -FR\left(1+\frac{3}{2}\sin\varphi_1\right)\cdot\left(-R(1+\sin\varphi_1+\cos\varphi_1)\right)\cdot R\,d\varphi_1 + \int_0^{\pi/2} -\frac{3}{2}FR\cdot\sin\varphi_2\cdot\left(-R(1+\sin\varphi_2-\cos\varphi_2)\right)\cdot R\,d\varphi_2\right] = \frac{FR^3}{EI_y}\left(\frac{1}{2}\cdot\frac{8}{3}+\frac{\pi}{2}+1+1+\frac{3}{2}\cdot 1+\frac{3}{2}\cdot\frac{\pi}{4}+\frac{3}{2}\cdot\frac{1}{2}+\frac{3}{2}\cdot 1+\frac{3}{2}\cdot\frac{\pi}{4}-\frac{3}{2}\cdot\frac{1}{2}\right) = \frac{FR^3}{EI_y}\left(\frac{19}{3}+\frac{5}{4}\pi\right) \approx 10{,}26\,\frac{FR^3}{EI_y}\;(\leftarrow) = u_C$$

Kut nagiba u osloncu B:

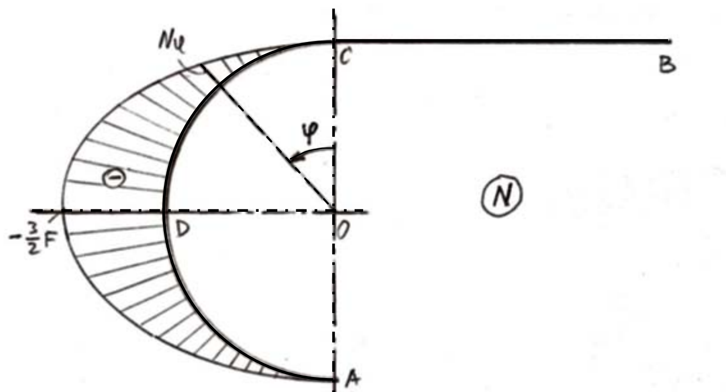
$$\alpha_B = \left(\frac{\partial U}{\partial M_B}\right)_{M_B=0} = \frac{F}{EI_y}\left[\int_0^{2R} -\frac{x}{2}\cdot\left(1-\frac{x}{2R}\right)dx + \int_0^{\frac{\pi}{2}} -R\left(1+\frac{3}{2}\sin\varphi_1\right)\left(-\frac{\sin\varphi_1}{2}\right)R\,d\varphi_1 + \int_0^{\frac{\pi}{2}} -\frac{3}{2}R\cdot\sin\varphi_2\cdot\left(-\frac{\sin\varphi_2}{2}\right)R\,d\varphi_2\right] = \frac{FR^2}{EI_y}\left[-\frac{1}{2}\cdot\frac{4}{2}+\frac{1}{4}\cdot\frac{8}{3}+\frac{1}{2}\cdot 1+\frac{3}{4}\cdot\frac{\pi}{4}+\frac{3}{4}\cdot\frac{\pi}{4}\right] = \frac{FR^2}{EI_y}\left(\frac{1}{6}+\frac{3\pi}{8}\right) \approx 1{,}345\,\frac{FR^2}{EI_y}$$

Kut nagiba u točki D:

$$\alpha_D = \left(\frac{\partial U}{\partial M_D}\right)_{M_D=0} = \frac{F}{EI_y}\left[\int_0^{2R} -\frac{x}{2}\cdot\left(-\frac{x}{2R}\right)dx + \int_0^{\pi/2} -R\left(1+\frac{3}{2}\sin\varphi_1\right)\cdot\left(-\left(1+\frac{\sin\varphi_1}{2}\right)\right)R\,d\varphi_1 + \int_0^{\pi/2} -\frac{3}{2}R\sin\varphi_2\cdot\left(-\frac{\sin\varphi_2}{2}\right)R\,d\varphi_2\right] =$$

$$= \frac{FR^2}{EI_y}\left(\frac{1}{\cancel{4}}\cdot\frac{\overset{2}{\cancel{8}}}{3} + \frac{\pi}{2} + \frac{1}{2}\cdot 1 + \frac{3}{2}\cdot 1 + \frac{3}{4}\cdot\frac{\pi}{4} + \frac{3}{4}\cdot\frac{\pi}{4}\right) = \frac{FR^2}{EI_y}\left(\frac{8}{3}+\frac{7}{8}\pi\right) \approx 5{,}4156\,\frac{FR^2}{EI_y}\ (G)$$

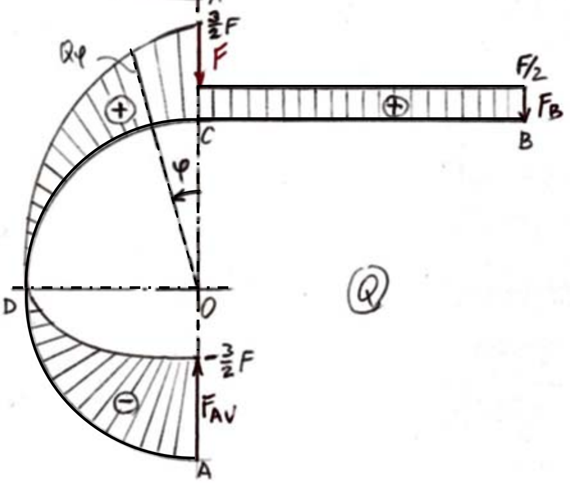
Dijagrami unutarnjih sila duž konture okvirnog nosača:



$$N_\varphi = -\frac{3}{2}F\cdot\sin\varphi\ ,\quad 0\le\varphi\le\pi$$

$$N_A = N_B = N_C = 0$$

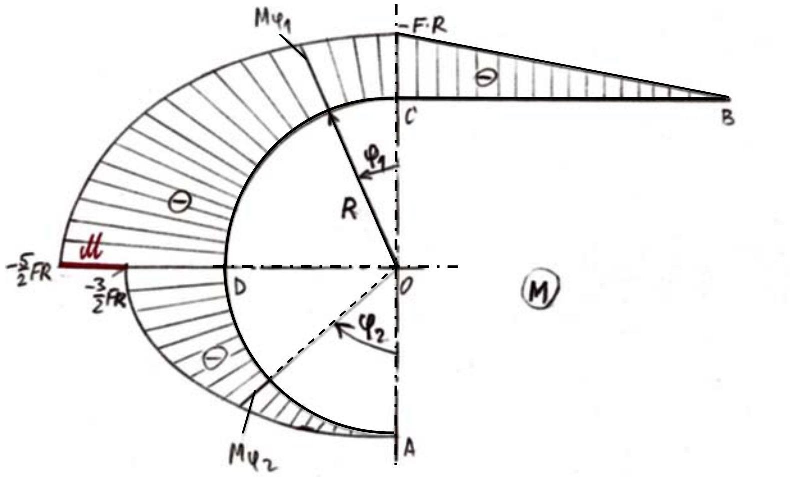
$$N_D = -\frac{3}{2}F$$



$$Q_\varphi = \frac{3}{2}F\cdot\cos\varphi\ ,\quad 0\le\varphi\le\pi$$

$$Q_B = \frac{F}{2}\ ,\ Q_A = -\frac{3}{2}F$$

$$Q_D = 0\ ,\ Q_{C,L} = \frac{3}{2}F$$



Momenti savijanja:

$$M_A = M_B = 0$$

$$M_C = -F_B\cdot 2R = -F\cdot R$$

$$M_{D,l} = -F_{AV}\cdot R = -\frac{3}{2}F\cdot R$$

$$M_{D,d} = M_{D,l} - \mathcal{M} = \left(-\frac{3}{2}-1\right)F\cdot R = -\frac{5}{2}F\cdot R$$

$$M_{\varphi_1} = -F_B\cdot(2R+R\cdot\sin\varphi_1) - F\cdot R\cdot\sin\varphi_1 = -F\cdot R\left(1+\frac{3}{2}\sin\varphi_1\right)\ ,\quad 0\le\varphi_1\le\frac{\pi}{2}$$

$$M_{\varphi_2} = -F_{AV}\cdot R\cdot\sin\varphi_2 = -\frac{3}{2}FR\cdot\sin\varphi_2\ ,\quad 0\le\varphi_2\le\frac{\pi}{2}$$

Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.