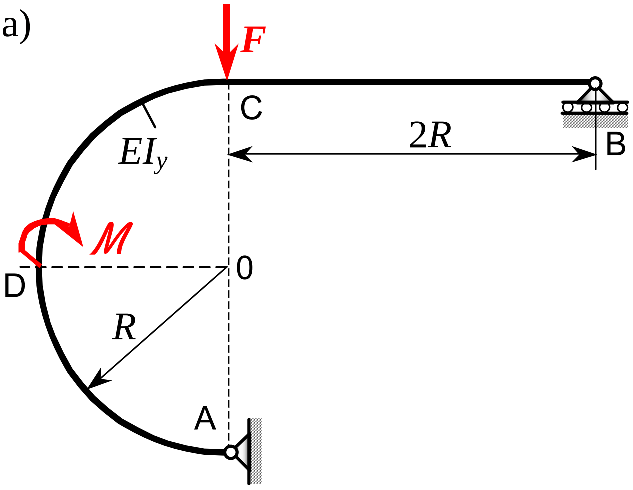
6. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

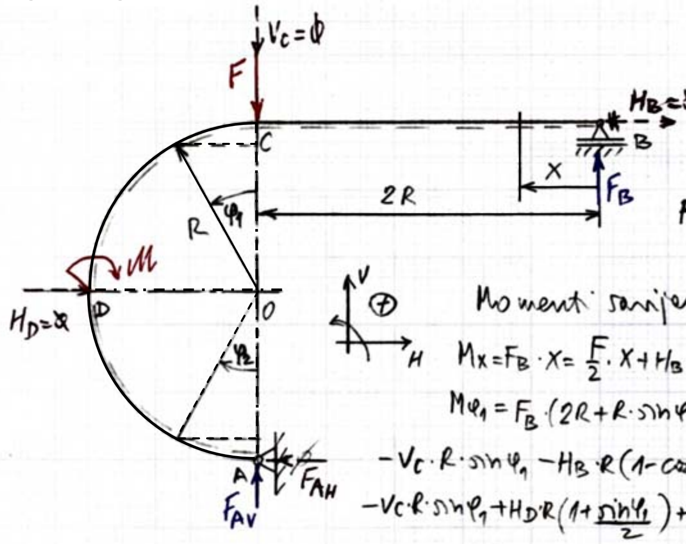
a)



Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba odrediti:
- reakcije veza u osloncima A i B
- vertikalni pomak u C (w_C = ?)
- vodoravne pomake u točkama B i D (u_B = ? i u_D = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: F, R, $\mathcal{M}=F\cdot R$, EI_y = konst.

Rješenje:



Jednadžbe ravnoteže:

1. $\Sigma F_H=\phi \quad -F_{AH}+H_B+H_D=\phi \rightarrow F_{AH}=H_B+H_D=\phi$
2. $\Sigma F_V=\phi \quad F_{AV}+F_B-F-V_C=\phi$
3. $\Sigma M_A=\phi \quad F_B\cdot 2R-\mathcal{M}-H_B\cdot 2R-H_D\cdot R=\phi \;/:2R$

$$F_B=\frac{F}{2}+H_B+\frac{H_D}{2},\quad F_{AV}=F+V_C-F_B=\frac{F}{2}+V_C-H_B-\frac{H_D}{2}$$

Momenti savijanja i derivacije:

$$M_x=F_B\cdot x=\frac{F}{2}\cdot x+H_B\cdot x+\frac{H_D}{2}\cdot x$$

$$M_{\varphi_1}=F_B(2R+R\sin\varphi_1)-F\cdot R\sin\varphi_1-V_C\cdot R\sin\varphi_1-H_B\cdot R(1-\cos\varphi_1)=FR\left(1-\frac{\sin\varphi_1}{2}\right)-V_C R\sin\varphi_1+H_D R\left(1+\frac{\sin\varphi_1}{2}\right)+H_B\cdot R(1+\sin\varphi_1+\cos\varphi_1)$$

$$M_{\varphi_2}=-F_{AV}R\sin\varphi_2+F_{AH}R(1-\cos\varphi_2)=-\frac{F}{2}R\sin\varphi_2-V_C R\sin\varphi_2+H_D R\left(1+\frac{\sin\varphi_2}{2}-\cos\varphi_2\right)+H_B\cdot R(1+\sin\varphi_2-\cos\varphi_2)$$

$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_D}$	$\frac{\partial M_i}{\partial H_B}$
ϕ	$\frac{x}{2}$	x
$-R\sin\varphi_1$	$R\left(1+\frac{\sin\varphi_1}{2}\right)$	$R(1+\sin\varphi_1+\cos\varphi_1)$
$-R\sin\varphi_2$	$R\left(1+\frac{\sin\varphi_2}{2}-\cos\varphi_2\right)$	$R(1+\sin\varphi_2-\cos\varphi_2)$

Vertikalni pomak u C:

$$w_C=\left(\frac{\partial U}{\partial V_C}\right)_{V_C=\phi}=\frac{FR^3}{EI_y}\left[\int_0^{\frac{\pi}{2}}\left(1-\frac{\sin\varphi_1}{2}\right)(-\sin\varphi_1)\,d\varphi_1+\int_0^{\frac{\pi}{2}}-\frac{\sin\varphi_2}{2}\cdot(-\sin\varphi_2)\,d\varphi_2\right]=\frac{FR^3}{EI_y}\left(-1+\frac{1}{2}\cdot\frac{\pi}{4}+\frac{1}{2}\cdot\frac{\pi}{4}\right)=\frac{FR^3}{EI_y}\left(\frac{\pi}{4}-1\right)\approx -0{,}2146\frac{FR^3}{EI_y}\ (\uparrow)$$

Vodoravni pomak u D:

$$u_D=\left(\frac{\partial U}{\partial H_D}\right)_{H_D=\phi}=\frac{F}{EI_y}\left[\int_0^{2R}\frac{x}{2}\cdot\frac{x}{2}\,dx+\int_0^{\pi/2}R\left(1-\frac{\sin\varphi_1}{2}\right)R\left(1+\frac{\sin\varphi_1}{2}\right)R\,d\varphi_1+\int_0^{\pi/2}-\frac{R}{2}\cdot\sin\varphi_2\cdot R\left(1+\frac{\sin\varphi_1}{2}-\cos\varphi_2\right)R\,d\varphi_2\right]=$$

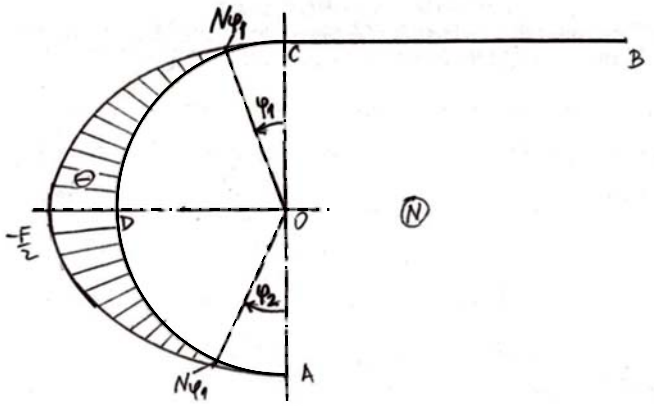
$$=\frac{FR^3}{EI_y}\left(\frac{1}{4}\cdot\frac{8}{3}+\frac{\pi}{2}-\frac{1}{4}\cdot\frac{\pi}{4}-\frac{1}{2}\cdot 1-\frac{1}{4}\cdot\frac{\pi}{4}+\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{FR^3}{EI_y}\left(\frac{11}{12}+\frac{3\pi}{8}\right)\approx 2{,}095\frac{FR^3}{EI_y}\ (\rightarrow)$$

Vodoravni pomak u B:

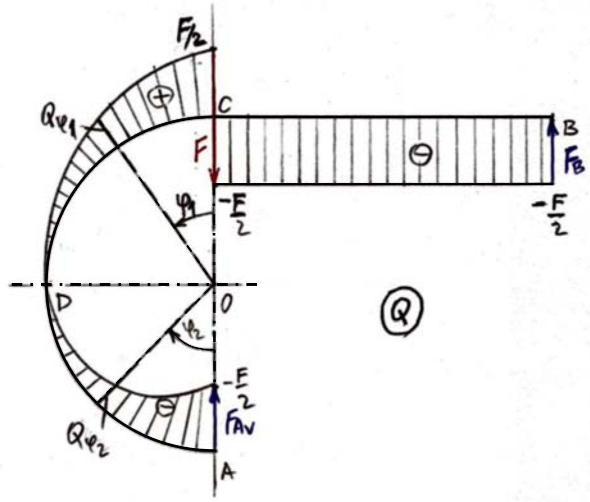
$$u_B=\left(\frac{\partial U}{\partial H_B}\right)_{H_B=\phi}=\frac{F}{EI_y}\left[\int_0^{2R}\frac{x}{2}\cdot x\,dx+\int_0^{\pi/2}R\left(1-\frac{\sin\varphi_1}{2}\right)\cdot R(1+\sin\varphi_1+\cos\varphi_1)R\,d\varphi_1+\int_0^{\pi/2}-\frac{R}{2}\cdot\sin\varphi_2\cdot R(1+\sin\varphi_2-\cos\varphi_2)R\,d\varphi_2\right]=$$

$$=\frac{FR^3}{EI_y}\left(\frac{1}{2}\cdot\frac{8}{3}+\frac{\pi}{2}+1+1-\frac{1}{2}\cdot 1-\frac{1}{2}\cdot\frac{\pi}{4}-\frac{1}{2}\cdot\frac{1}{2}-\frac{1}{2}\cdot 1-\frac{1}{2}\cdot\frac{\pi}{4}+\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{FR^3}{EI_y}\left(\frac{7}{3}+\frac{\pi}{4}\right)\approx 3{,}11873\frac{FR^3}{EI_y}\ (\rightarrow)$$

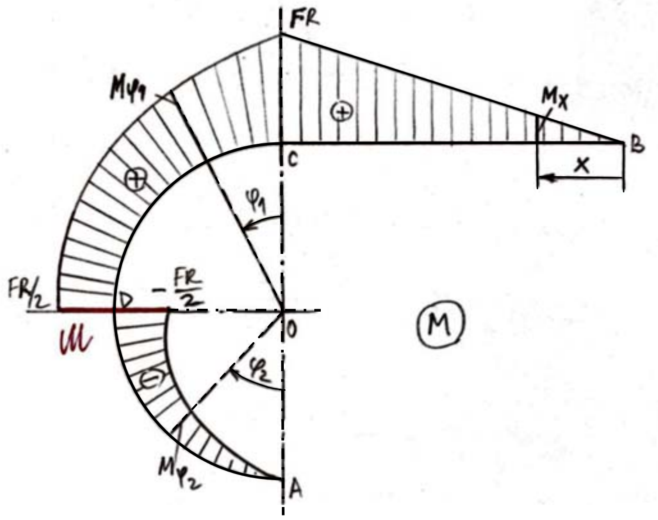
Dijagrami unutarnjih sila duž konture okvirnog nosača:



$$N_x = 0 \rightarrow N_B = N_C = 0$$
$$N\varphi_1 = -\frac{F}{2}\cdot\sin\varphi_1,\quad N_D = -\frac{F}{2},\quad 0 \le \varphi_1 \le \frac{\pi}{2}$$
$$N\varphi_2 = -\frac{F}{2}\cdot\sin\varphi_2,\quad N_A = 0,\quad 0 \le \varphi_2 \le \frac{\pi}{2}$$



$$Q_x = -\frac{F}{2} \rightarrow Q_B = Q_{C,D} = -\frac{F}{2}$$
$$Q_{C,L} = F - F_B = \frac{F}{2}$$
$$Q_{\varphi_1} = \frac{F}{2}\cdot\cos\varphi_1,\quad 0 \le \varphi_1 \le \frac{\pi}{2}$$
$$Q_D = 0$$
$$Q_{\varphi_2} = -\frac{F}{2}\cdot\cos\varphi_2,\quad 0 \le \varphi_2 \le \frac{\pi}{2}$$
$$Q_A = -\frac{F}{2}$$



$$M_x = F_B\cdot x = \frac{F\cdot x}{2},\quad M_B = 0,\quad M_C = FR$$
$$M_{\varphi_1} = FR\left(1 - \frac{1}{2}\cdot\sin\varphi_1\right),\quad M_{D,D} = \frac{1}{2}FR,\quad 0 \le \varphi_1 \le \frac{\pi}{2}$$
$$M_{D,L} = M_{D,D} - \mathcal{M} = -\frac{1}{2}FR$$
$$M_{\varphi_2} = -F_A\cdot R\cdot\sin\varphi_2 = -\frac{FR}{2}\cdot\sin\varphi_2,\quad 0 \le \varphi_2 \le \frac{\pi}{2}$$
$$M_A = 0$$

Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).