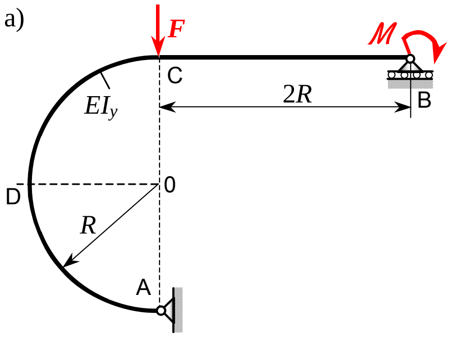
5. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

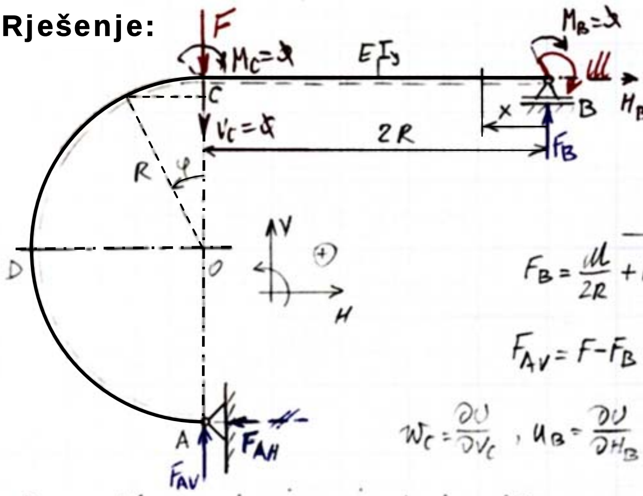


Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba odrediti:

- reakcije veza u osloncima A i B
- vertikalni pomak u C (w_C = ?)
- vodoravni pomak u točki B (u_B = ?)
- kutne zakrete na mjestima B i C (α_B = ?, α_C = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: $\boldsymbol{F}$, R, $\mathcal{M} = F \cdot R$, EI_y = konst.

Rješenje:



Reakcije u osloncima nosača:

1. $\Sigma F_H = 0 \quad -F_{AH} + H_B = 0 \rightarrow F_{AH} = H_B = 0$

2. $\Sigma F_V = 0 \quad F_{AV} + F_B - F - V_C = 0$

3. $\Sigma M_A = 0 \quad F_B \cdot 2R - \mathcal{M} - M_B - M_C - H_B \cdot 2R = 0 \;/ : 2R$

$$F_B = \frac{\mathcal{M}}{2R} + H_B + \frac{M_B}{2R} + \frac{M_C}{2R} = \frac{F}{2} + H_B + \frac{M_B}{2R} + \frac{M_C}{2R}, \qquad F_B = \frac{F}{2}$$

$$F_{AV} = F - F_B + V_C = \frac{F}{2} + V_C - H_B - \frac{M_B}{2R} - \frac{M_C}{2R}, \qquad F_{AV} = \frac{F}{2}$$

$$w_C = \frac{\partial U}{\partial V_C}, \quad u_B = \frac{\partial U}{\partial H_B}, \quad \alpha_B = \frac{\partial U}{\partial M_B}, \quad \alpha_C = \frac{\partial U}{\partial M_C}$$

Momenti savijanja i derivacije:

$$M_x = F_B \cdot x - \mathcal{M} - M_B = \frac{F}{2} \cdot x - F \cdot R + H_B \cdot x + \frac{M_B}{2R}x - M_B + \frac{M_C}{2R} \cdot x =$$
$$= F\left(\frac{x}{2} - R\right) + H_B \cdot x + M_B\left(\frac{x}{2R} - 1\right) + M_C \frac{x}{2R}$$

$$M_\varphi = F_B \cdot (2R + R \cdot \sin\varphi) - \mathcal{M} - M_B - H_B R(1 - \cos\varphi) - F \cdot R \cdot \sin\varphi -$$
$$- V_C \cdot R \cdot \sin\varphi - M_C = -\frac{F}{2} R \cdot \sin\varphi + H_B R(1 + \sin\varphi + \cos\varphi) -$$
$$- V_C \cdot R \cdot \sin\varphi + M_B \cdot \frac{\sin\varphi}{2} + M_C \cdot \frac{\sin\varphi}{2}$$

$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_B}$	$\frac{\partial M_i}{\partial M_B}$	$\frac{\partial M_i}{\partial M_C}$
0	x	$\left(\frac{x}{2R} - 1\right)$	$\frac{x}{2R}$
$-R \cdot \sin\varphi$	$R(1 + \sin\varphi + \cos\varphi)$	$+\frac{1}{2}\sin\varphi$	$+\frac{1}{2} \cdot \sin\varphi$

Vertikalni pomak u točki C:

$$w_C = \left(\frac{\partial U}{\partial V_C}\right)_{V_C=0} = \frac{1}{EI_y} \cdot \int_0^{\pi} \left(-\frac{F}{2} R \cdot \sin\varphi\right) \cdot (-R \cdot \sin\varphi) \cdot R\,d\varphi = \frac{FR^3}{EI_y} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{FR^3 \cdot \pi}{4EI_y} \approx 0{,}786 \cdot \frac{FR^3}{EI_y} \; (\downarrow)$$

Vodoravni pomak u B:

$$u_B = \left(\frac{\partial U}{\partial H_B}\right)_{H_B=0} = \frac{1}{EI_y}\left[\int_0^{2R} F\left(\frac{x}{2} - R\right) \cdot x\,dx + \int_0^{\pi} \left(-\frac{F}{2} \cdot R \cdot \sin\varphi\right) \cdot R(1 + \sin\varphi + \cos\varphi) R\,d\varphi\right] = \frac{FR^3}{EI_y}\left[\frac{1}{2} \cdot \frac{8}{3} - \frac{4}{2} - \frac{1}{2} \cdot 2 -\right.$$
$$\left. - \frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \cdot 0\right] = -\frac{FR^3}{EI_y}\left(\frac{5}{3} + \frac{\pi}{4}\right) \approx -2{,}452 \, \frac{FR^3}{EI_y} \; (\leftarrow)$$

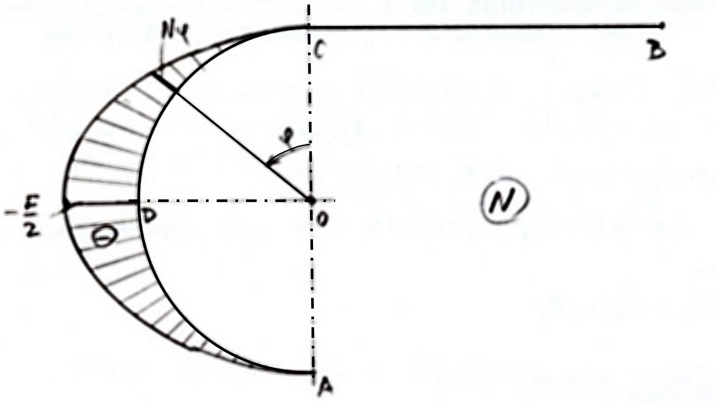
Kutni zakret u B:

$$\alpha_B = \left(\frac{\partial U}{\partial M_B}\right)_{M_B=0} = \frac{1}{EI_y}\left[\int_0^{2R} F\left(\frac{x}{2} - R\right)\left(\frac{x}{2R} - 1\right)dx + \int_0^{\pi} \left(-\frac{FR}{2}\sin\varphi\right) \cdot \left(+\frac{1}{2}\sin\varphi\right) R\,d\varphi\right] = \frac{FR^2}{EI_y}\left[\frac{1}{4} \cdot \frac{8}{3} - \frac{1}{2} \cdot \frac{4}{2} - \frac{1}{2} \cdot 4 +\right.$$
$$\left. + 2 - \frac{1}{4} \cdot \frac{\pi}{2}\right] = \frac{FR^2}{EI_y}\left(\frac{2}{3} - \frac{\pi}{8}\right) \approx 0{,}274 \cdot \frac{FR^2}{EI_y} \; (\circlearrowright)$$

Kutni zakret u C:

$$\alpha_C = \left(\frac{\partial U}{\partial M_C}\right)_{M_C=0} = \frac{1}{EI_y}\left[\int_0^{2R} F\left(\frac{x}{2} - R\right) \cdot \frac{x}{2R} \cdot dx + \int_0^{\pi} \left(-\frac{F}{2} R \cdot \sin\varphi\right) \cdot \frac{1}{2} \sin\varphi \cdot R\,d\varphi\right] = \frac{FR^2}{EI_y}\left[\frac{1}{4} \cdot \frac{8}{3} - \frac{1}{2} \cdot \frac{4}{2} - \frac{1}{4} \cdot \frac{\pi}{2}\right] =$$
$$= \frac{FR^2}{EI_y}\left(-\frac{1}{3} - \frac{\pi}{8}\right) = -\frac{FR^2}{EI_y}\left(\frac{1}{3} + \frac{\pi}{8}\right) \approx -0{,}728 \cdot \frac{FR^2}{EI_y} \; (\circlearrowleft)$$

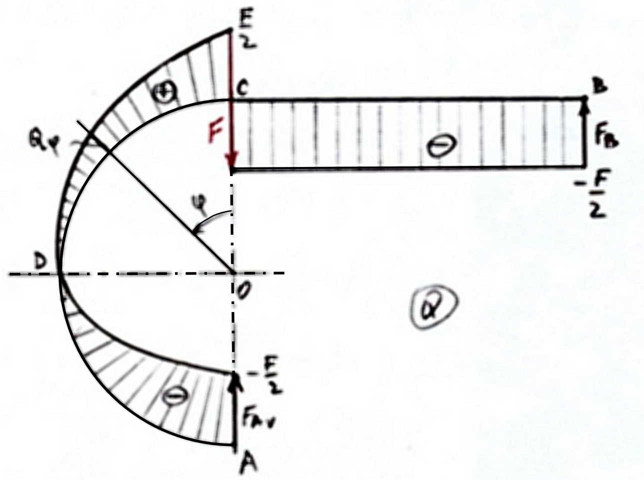
Dijagrami unutarnjih sila duž konture okvirnog nosača:



$$N_\varphi = -\frac{F}{2}\cdot\sin\varphi \quad , \quad 0 \le \varphi \le \pi$$

$$N_A = N_B = N_C = 0$$

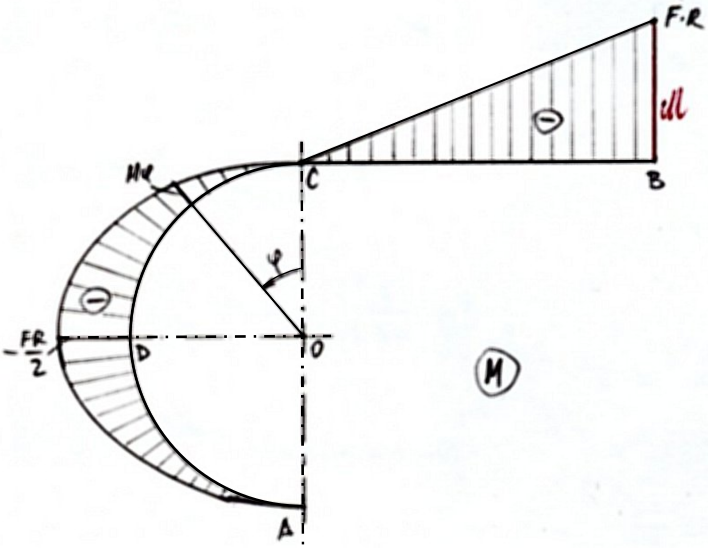
$$N_D = -F/2$$



$$Q_\varphi = \frac{F}{2}\cdot\cos\varphi \quad , \quad 0 \le \varphi \le \pi$$

$$Q_A = -\frac{F}{2} \ , \ Q_B = -\frac{F}{2} = Q_{C,D}$$

$$Q_{C,L} = \frac{F}{2} \ , \ Q_D = 0$$



$$M_A = 0 \ , \ M_B = -\mathcal{M} = -F\cdot R$$

$$M_C = F_B\cdot 2R - \mathcal{M} = \frac{F}{2}\cdot 2R - F\cdot R = 0$$

$$M_D = -F_{AV}\cdot R = -\frac{1}{2}F\cdot R$$

$$M_\varphi = F_B\cdot(2R + R\cdot\sin\varphi) - \mathcal{M} - F\cdot R\cdot\sin\varphi =$$
$$= -\frac{1}{2}F\cdot R\cdot\sin\varphi$$

Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).