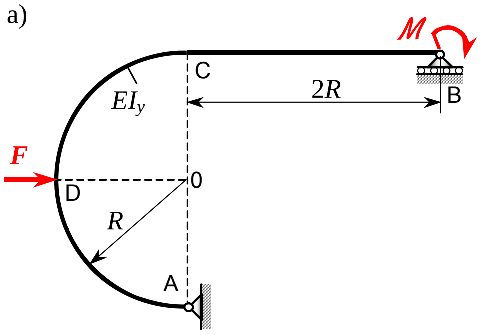
4. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

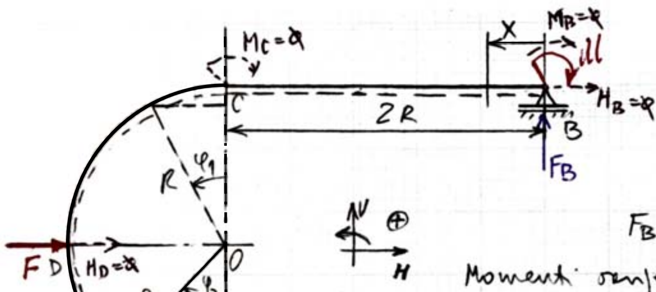


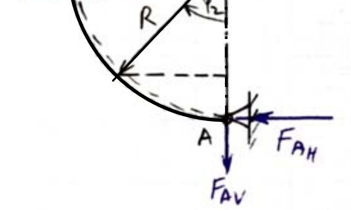
Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba odrediti:

- reakcije veza u osloncima A i B
- vodoravne pomake u točkama B i D (u_B = ?, u_D = ?)
- kutne zakrete na mjestima B i C (α_B = ?, α_C = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: F, R, $M = F\cdot R$, EI_y = konst.

Rješenje:





Jedn. ravnoteže:

1. $\Sigma F_H = 0 \quad F - F_{AH} + H_B + H_D = 0$

$F_{AH} = F + H_B + H_D \quad , \quad F_{AH} = F$

2. $\Sigma F_V = 0 \quad -F_{AV} + F_B = 0 \rightarrow F_{AV} = F_B = F$

3. $\Sigma M_A = 0 \quad F_B\cdot 2R - F\cdot R - M - H_B\cdot 2R - H_D\cdot R - M_B - M_C = 0 \quad /:2R$

$$F_B = F + H_B + \frac{H_D}{2} + \frac{M_B}{2R} + \frac{M_C}{2R} = F_{AV}$$

Momenti savijanja i derivacije za pomake i kutove:

$$M_x = F_B\cdot x - M - M_B = F\cdot x + H_B\cdot x + \frac{H_D}{2}\cdot x + \frac{M_B}{2R}\cdot x + \frac{M_C}{2R}\cdot x - F\cdot R - M_B = F\cdot x - F\cdot R$$

$$M_{\varphi_1} = F_B(2R + R\cdot\sin\varphi_1) - M - H_B\cdot R(1-\cos\varphi_1) - M_B - M_C = F R(2+\sin\varphi_1) + H_B\cdot R(2+\sin\varphi_1) + \frac{H_D}{2}\cdot R\cdot(2+\sin\varphi_1) + \frac{M_B}{2}(2+\sin\varphi_1) + \frac{M_C}{2}\cdot(2+\sin\varphi_1) - M - H_B\cdot R(1-\cos\varphi_1) - M_B - M_C = F\cdot R(1+\sin\varphi_1)$$

$$M_{\varphi_2} = F_{AH}\cdot R(1-\cos\varphi_2) + F_{AV}\cdot R\cdot\sin\varphi_2 = F\cdot R\cdot(1-\cos\varphi_2) + H_B\cdot R(1-\cos\varphi_2) + H_D\cdot R(1-\cos\varphi_2) + F\cdot R\cdot\sin\varphi_2 + H_B\cdot R\cdot\sin\varphi_2 + \frac{H_D}{2}\cdot R\cdot\sin\varphi_2 + \frac{M_B}{2}\cdot\sin\varphi_2 + \frac{M_C}{2}\cdot\sin\varphi_2 = F\cdot R(1+\sin\varphi_2-\cos\varphi_2)$$

$\frac{\partial M_i}{\partial H_B}$	$\frac{\partial M_i}{\partial H_D}$	$\frac{\partial M_i}{\partial M_B}$	$\frac{\partial M_i}{\partial M_C}$
x	$\frac{x}{2}$	$\frac{x}{2R}-1$	$\frac{x}{2R}$
$(1+\sin\varphi_1+\cos\varphi_1)\cdot R$	$R(1+\frac{\sin\varphi_1}{2})$	$\frac{1}{2}\sin\varphi_1$	$\frac{1}{2}\sin\varphi_1$
$R(1+\sin\varphi_2-\cos\varphi_2)$	$R(1+\frac{\sin\varphi_2}{2}-\cos\varphi_2)$	$\frac{1}{2}\sin\varphi_2$	$\frac{1}{2}\sin\varphi_2$

Vodoravni pomak u B i D:

$$u_B = \left(\frac{\partial U}{\partial H_B}\right)_{H_B=0} = \frac{1}{EI_y}\left[\int_0^{2R} F(x-R)\cdot x\,dx + \int_0^{\pi/2} F\cdot R\cdot(1+\sin\varphi_1)\cdot R(1+\sin\varphi_1+\cos\varphi_1)\cdot R\,d\varphi_1 + \int_0^{\pi/2} F\cdot R^2(1+\sin\varphi_2-\cos\varphi_2)^2 R\,d\varphi_2\right] =$$

$$= \frac{FR^3}{EI_y}\left[\frac{8}{3} - \cancel{2} + \frac{\pi}{2} + \cancel{1} + 1 + \cancel{1} + \frac{\pi}{4} + \cancel{\frac{1}{2}} + \frac{\pi}{2} + \cancel{1} - 1 + 1 + \frac{\pi}{4} - \cancel{\frac{1}{2}} - \cancel{1} - \frac{1}{2} + \frac{\pi}{4}\right] = \frac{FR^3}{EI_y}\left(\frac{8}{3} + \frac{1}{2} + \pi + \frac{3\pi}{4}\right) =$$

$$= \frac{FR^3}{EI_y}\left(\frac{19}{6} + \frac{7}{4}\pi\right) \approx 8{,}6645\cdot\frac{FR^3}{EI_y} \; (\rightarrow)$$

$$u_D = \left(\frac{\partial U}{\partial H_D}\right)_{H_D=0} = \frac{1}{EI_y}\left[\int_0^{2R} F(x-R)\cdot\frac{x}{2}\,dx + \int_0^{\pi/2} FR(1+\sin\varphi_1)\cdot R\left(1+\frac{\sin\varphi_1}{2}\right)R\,d\varphi_1 + \int_0^{\pi/2} F\cdot R(1+\sin\varphi_2-\cos\varphi_2)\cdot R\left(1+\frac{\sin\varphi_2}{2}-\cos\varphi_2\right)R\,d\varphi_2\right]$$

$$= \frac{FR^3}{EI_y}\left[\frac{1}{2}\cdot\frac{8}{3} - \frac{1}{2}\cdot\frac{4}{2} + \frac{\pi}{2} + \frac{1}{2}\cdot 1 + 1 + \frac{1}{2}\cdot\frac{\pi}{4} + \frac{\pi}{2} + \frac{1}{2}\cdot 1 - 1 + 1 + \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{2} - 1 - \frac{1}{2}\cdot\frac{1}{2} + \frac{\pi}{4}\right] = \frac{FR^3}{EI_y}\left(\frac{8}{6} - \frac{3}{4} + \pi + \frac{\pi}{2}\right) =$$

$$= \frac{FR^3}{EI_y}\left(\frac{7}{12} + \frac{3}{2}\pi\right) \approx 5{,}2857\cdot\frac{FR^3}{EI_y} \; (\rightarrow)$$

Kutni zakret u B i C:

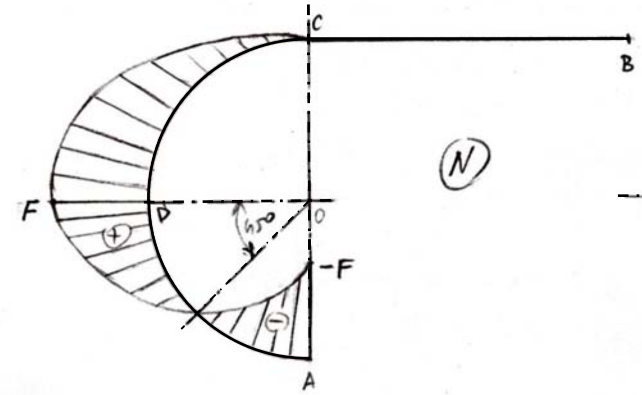
$$\alpha_C = \left(\frac{\partial U}{\partial M_C}\right)_{M_C=0} = \frac{1}{EI_y}\left[\int_0^{2R} F(x-R)\cdot\frac{x}{2R}\,dx + \int_0^{\pi/2} F\cdot R(1+\sin\varphi_1)\cdot\frac{1}{2}\sin\varphi_1\cdot R\,d\varphi_1 + \int_0^{\pi/2} FR(1+\sin\varphi_2-\cos\varphi_2)\cdot\frac{1}{2}\sin\varphi_2\, R\,d\varphi_2\right] =$$

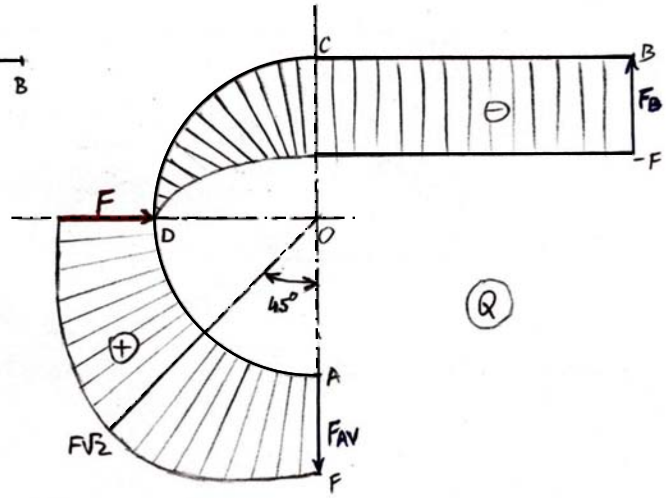
$$= \frac{FR^2}{EI_y}\left[\frac{8}{6} - 1 + \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{\pi}{4} + \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{2}\cdot\frac{1}{2}\right] = \frac{FR^2}{EI_y}\left(\frac{13}{12} + \frac{\pi}{4}\right) \approx 1{,}8687\,\frac{FR^2}{EI_y} \; (\circlearrowright)$$

$$\alpha_B = \left(\frac{\partial U}{\partial M_B}\right)_{M_B=\mathcal{M}} = \frac{1}{EI_y}\left[\int_0^{2R} F(x-R)\cdot\left(\frac{x}{2R}-1\right)dx + \int_0^{\frac{\pi}{2}} F\cdot R(1+\sin\varphi_1)\cdot\frac{1}{2}\sin\varphi_1\cdot R\,d\varphi_1 + \int_0^{\frac{\pi}{2}} F\cdot R(1+\sin\varphi_2-\cos\varphi_2)\cdot\frac{\sin\varphi_2}{2}R\,d\varphi_2\right]=$$

$$=\frac{FR^2}{EI_y}\left[\frac{8}{6}-1-\cancel{2}+\cancel{2}+\frac{1}{2}\cdot 1+\frac{1}{2}\cdot\frac{\pi}{4}+\frac{1}{2}\cdot 1+\frac{1}{2}\cdot\frac{\pi}{4}-\frac{1}{2}\cdot\frac{1}{2}\right]=\frac{FR^2}{EI_y}\left(\frac{13}{12}+\frac{\pi}{4}\right)\cong 1{,}8687\,\frac{FR^2}{EI_y} \quad (2)$$

Dijagrami unutarnjih sila duž konture okvirnog nosača:



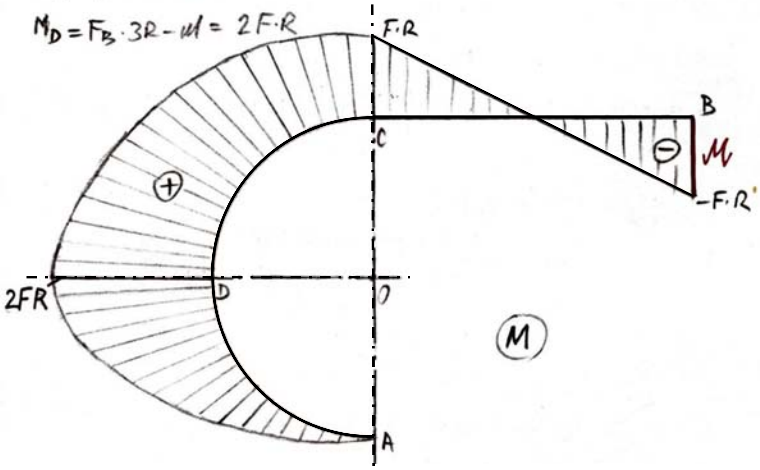


Momenti savijanja:

$M_B = -\mathcal{M} = -F\cdot R\,,\quad M_A = 0$

$M_C = F_B\cdot 2R - \mathcal{M} = F\cdot R$

$M_D = F_B\cdot 3R - \mathcal{M} = 2F\cdot R$



Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).

