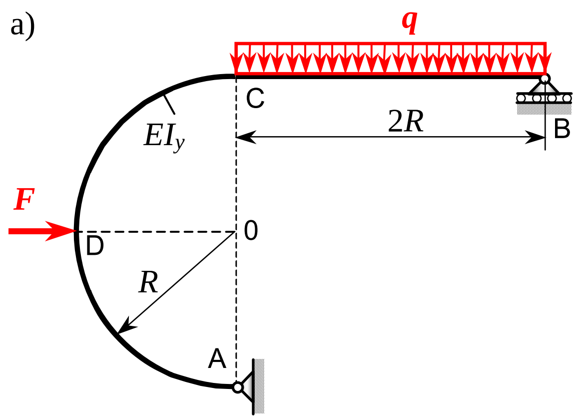
3. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

a)

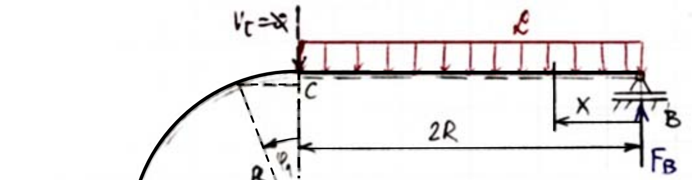


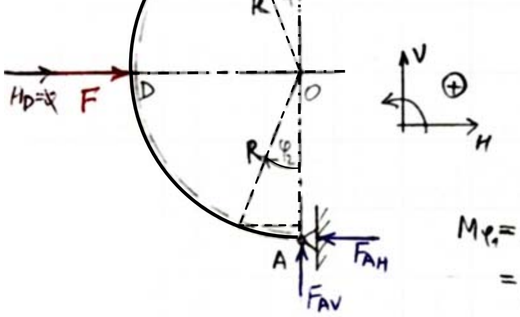
Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba:

- odrediti reakcije veza u osloncima A i B
- odrediti vertikalni pomak u C (w_C = ?)
- odrediti vodoravni pomak u točki D (u_D = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: $\boldsymbol{q}$, R, $\boldsymbol{F} = q \cdot R$, EI_y = konst.

Rješenje:





Jednadžbe ravnoteže:

1. $\Sigma F_H = 0 \quad F + H_D - F_{AH} = 0 \rightarrow F_{AH} = qR + H_D$
2. $\Sigma F_V = 0 \quad F_{AV} + F_B - q \cdot 2R - V_C = 0$
3. $\Sigma M_A = 0 \quad F_B \cdot 2R - q \cdot 2R \cdot R - F \cdot R - H_D \cdot R = 0 \;/ : 2R$

$$F_B = \frac{3}{2}qR + \frac{H_D}{2}, \quad F_{AV} = 2qR - F_B + V_C = \frac{1}{2}qR - \frac{H_D}{2} + V_C$$

Momentiranje i derivacije:

$$M_x = F_B \cdot x - \frac{qx^2}{2} = \frac{3}{2}qR \cdot x - \frac{qx^2}{2} + H_D \cdot \frac{x}{2}$$

$$M_{\varphi_1} = F_B(2R + R \cdot \sin\varphi_1) - 2qR \cdot (R + R \cdot \sin\varphi_1) - V_C R \cdot \sin\varphi_1 = qR^2\left(1 - \frac{\sin\varphi_1}{2}\right) - V_C R \cdot \sin\varphi_1 + H_D R\left(1 + \frac{\sin\varphi_1}{2}\right)$$

$$M_{\varphi_2} = -F_{AV} \cdot R \sin\varphi_2 + F_{AH} \cdot R(1 - \cos\varphi_2) = -\frac{qR^2}{2} \cdot \sin\varphi_2 + H_D R \cdot \frac{\sin\varphi_2}{2} - V_C \cdot R \cdot \sin\varphi_2 + qR^2(1 - \cos\varphi_2) + H_D \cdot R(1 - \cos\varphi_2) = qR^2\left(1 - \frac{\sin\varphi_2}{2} - \cos\varphi_2\right) + H_D \cdot R\left(1 + \frac{\sin\varphi_2}{2} - \cos\varphi_2\right) - V_C \cdot R \cdot \sin\varphi_2$$

	$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_D}$
M_x	0	$\frac{x}{2}$
M_{φ_1}	$-R \cdot \sin\varphi_1$	$R\left(1 + \frac{\sin\varphi_1}{2}\right)$
M_{φ_2}	$-R \sin\varphi_2$	$R\left(1 + \frac{\sin\varphi_2}{2} - \cos\varphi_2\right)$

Vertikalni pomak u C:

$$w_C = \left(\frac{\partial U}{\partial V_C}\right)_{V_C=0} = \frac{qR^2}{EI_y}\left[\int_0^{\frac{\pi}{2}}\left(1 - \frac{\sin\varphi_1}{2}\right) \cdot (-R \cdot \sin\varphi_1) R\, d\varphi_1 + \int_0^{\pi/2}\left(1 - \frac{\sin\varphi_2}{2} - \cos\varphi_2\right) \cdot (-R \cdot \sin\varphi_2) R\, d\varphi_2\right] =$$

$$= \frac{qR^4}{EI_y}\left(-1 + \frac{1}{2} \cdot \frac{\pi}{4} - 1 + \frac{1}{2} \cdot \frac{\pi}{4} + \frac{1}{2}\right) = \frac{qR^4}{EI_y}\left(\frac{\pi}{4} - \frac{3}{2}\right) \approx -0{,}7146 \cdot \frac{qR^4}{EI_y} \; (\uparrow)$$

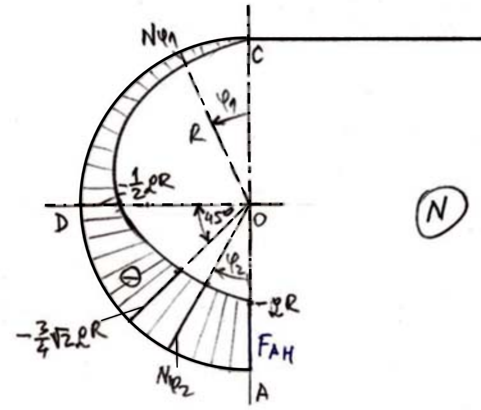
Vodoravni pomak u D:

$$u_D = \left(\frac{\partial U}{\partial H_D}\right)_{H_D=0} = \frac{q}{EI_y}\left[\int_0^{2R}\left(\frac{3}{2}R x - \frac{x^2}{2}\right) \cdot \frac{x}{2}\, dx + \int_0^{\pi/2} R^2\left(1 - \frac{\sin\varphi_1}{2}\right) \cdot R \cdot \left(1 + \frac{\sin\varphi_1}{2}\right) R\, d\varphi_1 + \int_0^{\pi/2} R^2\left(1 - \frac{\sin\varphi_2}{2} - \cos\varphi_2\right) \cdot R\left(1 + \frac{\sin\varphi_2}{2} - \cos\varphi_2\right) \cdot R\, d\varphi_2\right] =$$

$$= \frac{qR^4}{EI_y}\left[\frac{3}{4} \cdot \frac{8}{3} - \frac{1}{4} \cdot 4 + \frac{\pi}{2} - \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 - \frac{1}{4} \cdot \frac{\pi}{4} + \frac{\pi}{2} - \frac{1}{2} \cdot 1 - 1 + \frac{1}{2} \cdot 1 - \frac{1}{4} \cdot \frac{\pi}{4} - \frac{1}{2} \cdot \frac{1}{2} - 1 + \frac{1}{2} \cdot \frac{1}{2} + \frac{\pi}{4}\right] = \frac{qR^4}{EI_y}\left(\pi + \frac{\pi}{4} - \frac{\pi}{8} - 1\right) = \frac{qR^4}{EI_y}\left(\frac{9}{8}\pi - 1\right) \approx 2{,}5343 \cdot \frac{qR^4}{EI_y} \; (\rightarrow)$$

Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

Dijagrami unutarnjih sila duž konture okvirnog nosača:



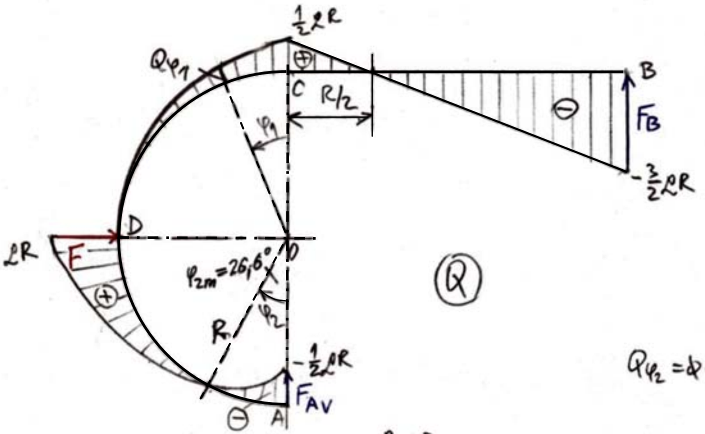
$N_x = \phi$, $N_{\varphi_2} = -F_{AH}\cdot\cos\varphi_2 - F_{AV}\cdot\sin\varphi_2 =$
$= -qR(\cos\varphi_2 + \frac{1}{2}\cdot\sin\varphi_2)$, $0 \le \varphi_2 \le \frac{\pi}{2}$

$N(\varphi_2 = 45°) = -\frac{3\sqrt{2}}{4}qR \cong -1{,}06\,qR$

$N_{\varphi_1} = -\frac{1}{2}qR\cdot\sin\varphi_1$, $0 \le \varphi_1 \le \frac{\pi}{2}$

$N(\varphi_{2m}) = -1{,}118\,qR$

$\varphi_{2m} = 26{,}6°$



$Q_x = -F_B + q\cdot x = -\frac{3}{2}qR + q\cdot x$, $0 \le x \le 2R$

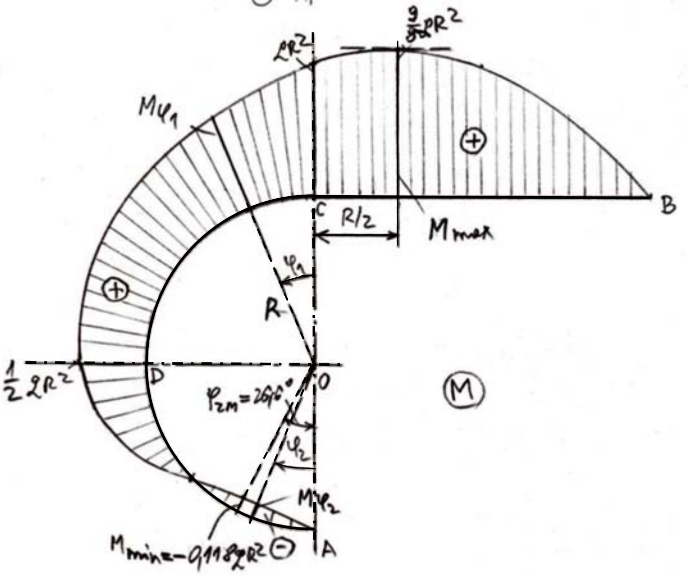
$Q_B = -\frac{3}{2}qR$, $Q_C = \frac{1}{2}qR$, $Q = 0$ za $x = \frac{3}{2}R$

$Q_{\varphi_1} = \frac{1}{2}qR\cdot\cos\varphi_1$, $0 \le \varphi_1 \le \frac{\pi}{2}$

$Q_{D,D} = 0$

$Q_{\varphi_2} = -F_{AV}\cdot\cos\varphi_2 + F_{AH}\cdot\sin\varphi_2 =$
$= qR(-\frac{1}{2}\cdot\cos\varphi_2 + \sin\varphi_2)$, $0 \le \varphi_2 \le \frac{\pi}{2}$

$Q_{\varphi_2} = 0 \to \sin\varphi_2 = \frac{1}{2}\cos\varphi_2 \,/ : \cos\varphi_2 \to \tan\varphi_2 = \frac{1}{2} \to \varphi_{2m} \cong 26{,}6°$



$M_x = \frac{3}{2}qR\cdot x - \frac{q x^2}{2}$, $0 \le x \le 2R$

$M_B = \phi$, $M_C = \frac{3}{2}qR\cdot 2R - \frac{q\cdot 4R^2}{2} = qR^2$

$M_{max} = \frac{3}{2}qR\cdot\frac{3}{2}R - \frac{q}{2}\cdot\frac{9}{4}R^2 = \frac{9}{8}qR^2$

$M_{\varphi_1} = qR^2(1 - \frac{1}{2}\sin\varphi_1)$, $0 \le \varphi_1 \le \frac{\pi}{2}$

$M_D = \frac{1}{2}qR^2$

$M_{\varphi_2} = qR^2(1 - \frac{1}{2}\sin\varphi_2 - \cos\varphi_2)$, $0 \le \varphi_2 \le \frac{\pi}{2}$

za $\varphi_2 = 26{,}6° \to M_{\varphi_2} \cong -0{,}118\cdot qR^2 = M_{min}$

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).