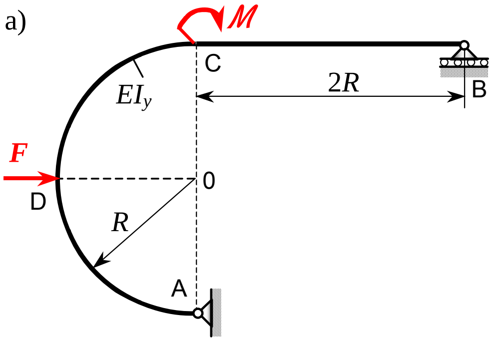
2. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

a)

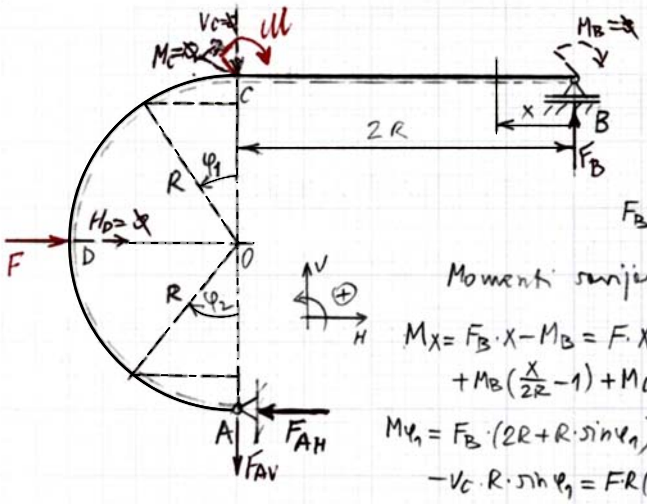


Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba odrediti:

- reakcije veza u osloncima A i B
- vertikalni pomak u točki C (w_C = ?)
- vodoravni pomak u točki D (u_D = ?)
- kutne zakrete u točkama B (α_B = ?) i C (α_C = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: $\boldsymbol{F}$, R, $\mathcal{M} = F \cdot R$, EI_y = konst.

Rješenje:



Jedn. ravnoteže:

1. $\Sigma F_H = 0 \quad F_{AH} = F + H_D$
2. $\Sigma F_V = 0 \quad -F_{AV} + F_B - V_C = 0$
3. $\Sigma M_A = 0 \quad -F \cdot R - \mathcal{M} + F_B \cdot 2R - H_D \cdot R - M_B - M_C = 0 \;/ : 2R$

$$F_B = F + \frac{H_D}{2} + \frac{M_B}{2R} + \frac{M_C}{2R}, \quad F_{AV} = F - V_C + \frac{H_D}{2} + \frac{M_B}{2R} + \frac{M_C}{2R}$$

Momenti savijanja i deriviranje:

$$M_x = F_B \cdot x - M_B = F \cdot x + H_D \cdot \frac{x}{2} + M_B\left(\frac{x}{2R} - 1\right) + M_C \cdot \frac{x}{2R}$$

$$M_{\varphi_1} = F_B \cdot (2R + R \cdot \sin\varphi_1) - \mathcal{M} - M_B - M_C - V_C \cdot R \cdot \sin\varphi_1 = FR(1 + \sin\varphi_1) + M_B \cdot \frac{\sin\varphi_1}{2} + H_D R\left(1 + \frac{\sin\varphi_1}{2}\right) + M_C \cdot \frac{\sin\varphi_1}{2} - V_C R \cdot \sin\varphi_1$$

$$M_{\varphi_2} = F_{AV} \cdot R \cdot \sin\varphi_2 + F_{AH} \cdot R(1 - \cos\varphi_2) = F \cdot R \cdot (1 + \sin\varphi_2 - \cos\varphi_2) + \frac{M_C}{2} \sin\varphi_2 + H_D \cdot R\left(1 + \frac{\sin\varphi_2}{2} - \cos\varphi_2\right) + M_B \cdot \frac{\sin\varphi_2}{2} - V_C \cdot R \cdot \sin\varphi_2$$

$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_D}$	$\frac{\partial M_i}{\partial M_B}$	$\frac{\partial M_i}{\partial M_C}$
0	$\frac{x}{2}$	$\left(\frac{x}{2R} - 1\right)$	$\frac{x}{2R}$
$-R \cdot \sin\varphi_1$	$R\left(1 + \frac{\sin\varphi_1}{2}\right)$	$\frac{\sin\varphi_1}{2}$	$\frac{\sin\varphi_1}{2}$
$-R \cdot \sin\varphi_2$	$R\left(1 + \frac{\sin\varphi_2}{2} - \cos\varphi_2\right)$	$\frac{\sin\varphi_2}{2}$	$\frac{\sin\varphi_2}{2}$

Vertikalni pomak točke C:

$$w_C = \left(\frac{\partial U}{\partial V_C}\right)_{V_C = 0} = \frac{F}{EI_y}\left[\int_0^{\frac{\pi}{2}} R(1 + \sin\varphi_1)(-R \cdot \sin\varphi_1) R\, d\varphi_1 + \int_0^{\frac{\pi}{2}} R(1 + \sin\varphi_2 - \cos\varphi_2) \cdot (-R \cdot \sin\varphi_2) R\, d\varphi_2\right] = \frac{FR^3}{EI_y}\left(-1 - \frac{\pi}{4} - 1 - \frac{\pi}{4} + \frac{1}{2}\right) =$$

$$= -\frac{FR^3}{EI_y}\left(\frac{3}{2} + \frac{\pi}{2}\right) \approx -3{,}071 \frac{FR^3}{EI_y} (\uparrow)$$

Vodoravni pomak točke D:

$$u_D = \left(\frac{\partial U}{\partial H_D}\right)_{H_D = 0} = \frac{F}{EI_y}\left[\int_0^{2R} x \cdot \frac{x}{2}\, dx + \int_0^{\pi/2} R(1 + \sin\varphi_1) \cdot R\left(1 + \frac{\sin\varphi_1}{2}\right) R\, d\varphi_1 + \int_0^{\pi/2} R(1 + \sin\varphi_2 - \cos\varphi_2) \cdot R\left(1 + \frac{\sin\varphi_2}{2} - \cos\varphi_2\right) R\, d\varphi_2\right]$$

$$= \frac{FR^3}{EI_y}\left(\frac{1}{\cancel{2}} \cdot \frac{\cancel{8}^4}{3} + \frac{\pi}{2} + \frac{1}{2} \cdot 1 + \cancel{1} + \frac{1}{2} \cdot \frac{\pi}{4} + \frac{\pi}{2} + \frac{1}{\cancel{2}} \cdot 1 - \cancel{1} + 1 + \frac{1}{2} \cdot \frac{\pi}{4} - \cancel{\frac{1}{2}} - 1 - \frac{1}{2} \cdot \frac{1}{2} + \frac{\pi}{4}\right) = \frac{FR^3}{EI_y}\left(\frac{19}{12} + \frac{3\pi}{2}\right) \approx 6{,}296 \frac{FR^3}{EI_y}$$

$(\rightarrow)$

Kutni zakret u točki B:

$$\alpha_B = \left(\frac{\partial U}{\partial M_B}\right)_{M_B = 0} = \frac{F}{EI_y}\left[\int_0^{2R} x \cdot \left(\frac{x}{2R} - 1\right) dx + \int_0^{\frac{\pi}{2}} R(1 + \sin\varphi_1) \cdot \frac{\sin\varphi_1}{2} R\, d\varphi_1 + \int_0^{\frac{\pi}{2}} R(1 + \sin\varphi_2 - \cos\varphi_2) \cdot \frac{\sin\varphi_2}{2} R\, d\varphi_2\right] =$$

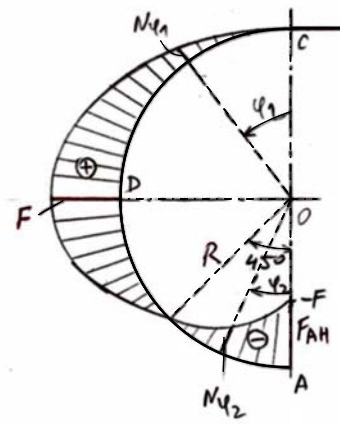
$$= \frac{FR^2}{EI_y}\left(\frac{1}{\cancel{2}} \cdot \frac{\cancel{8}^4}{3} - \frac{4^2}{2} + \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{\pi}{4} + \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{FR^2}{EI_y}\left(\frac{1}{12} + \frac{\pi}{4}\right) \approx 0{,}869 \frac{FR^2}{EI_y} (\curvearrowright)$$

Kutni zakret u točki C:

$$\alpha_C = \left(\frac{\partial U}{\partial M_C}\right)_{M_C = 0} = \frac{F}{EI_y}\left[\int_0^{2R} x \cdot \frac{x}{2R}\, dx + \int_0^{\frac{\pi}{2}} R(1 + \sin\varphi_1) \cdot \frac{\sin\varphi_1}{2} R\, d\varphi_1 + \int_0^{\frac{\pi}{2}} R(1 + \sin\varphi_2 - \cos\varphi_2) \frac{\sin\varphi_2}{2} \cdot R\, d\varphi_2\right] =$$

$$= \frac{FR^2}{EI_y}\left(\frac{1}{\cancel{2}} \cdot \frac{\cancel{8}^4}{3} + \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{\pi}{4} + \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{\pi}{4} - \frac{1}{2} \cdot \frac{1}{2}\right) = \frac{FR^2}{EI_y}\left(\frac{25}{12} + \frac{\pi}{4}\right) \approx 2{,}863 \frac{FR^2}{EI_y} (\curvearrowright)$$

Dijagrami unutarnjih sila duž konture okvirnog nosača:



$N_A = -F_{AH} = -F$, $N_B = N_C = 0$

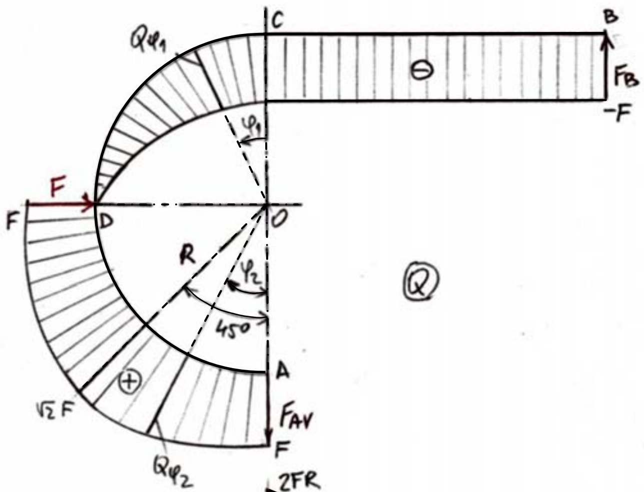
$N_D = F_B = F$

$N_{\varphi_1} = F_B \cdot \sin\varphi_1 = F \cdot \sin\varphi_1$, $0 \le \varphi_1 \le \frac{\pi}{2}$

$N_{\varphi_2} = F_{AV} \cdot \sin\varphi_2 - F_{AH} \cdot \cos\varphi_2 =$

$= F(\sin\varphi_2 - \cos\varphi_2)$, $0 \le \varphi_2 \le \frac{\pi}{2}$

$N = 0$ za $\varphi_2 = 45°$.



$Q_A = F_{AV} = F$, $Q_B = -F_B = -F = Q_C$

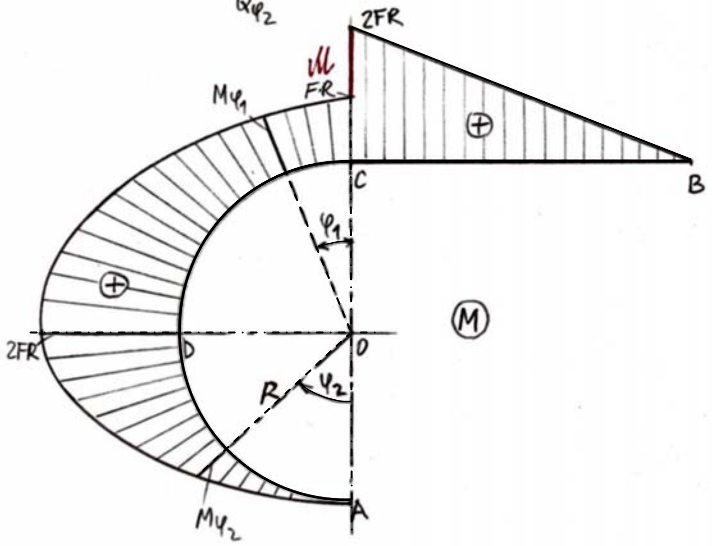
$Q_{\varphi_1} = -F \cdot \cos\varphi_1$, $0 \le \varphi_1 \le \frac{\pi}{2}$

$Q_{\varphi_2} = F_{AV} \cdot \cos\varphi_2 + F_{AH} \cdot \sin\varphi_2 = F(\cos\varphi_2 + \sin\varphi_2)$

$0 \le \varphi_2 \le \frac{\pi}{2}$

$Q_{max} = \sqrt{2} \cdot F$ za $\varphi_2 = 45°$.

$Q_D(\varphi_2) = F$, $Q_D(\varphi_1) = 0$.



Momenti savijanja:

$M_A = M_B = 0$

$M_{C,D} = F_B \cdot 2R = 2FR$, $M_{C,L} = M_{C,D} - \mathfrak{M} = FR$

$M_{\varphi_1} = FR(1 + \sin\varphi_1)$, $0 \le \varphi_1 \le \frac{\pi}{2}$

$M_{\varphi_2} = FR(1 + \sin\varphi_2 - \cos\varphi_2)$, $0 \le \varphi_2 \le \frac{\pi}{2}$

$M_D = 2FR$

Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).