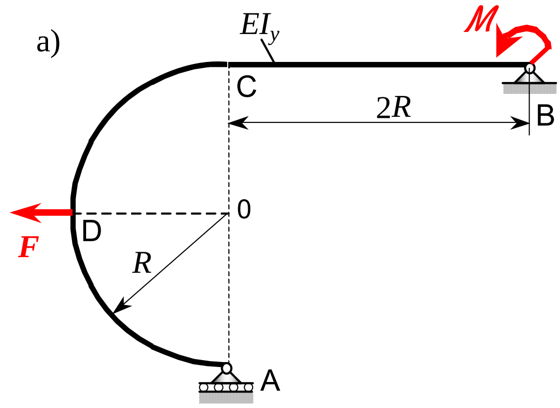
1. Zadatak: Izračunavanje deformacija za ravninski okvirni nosač

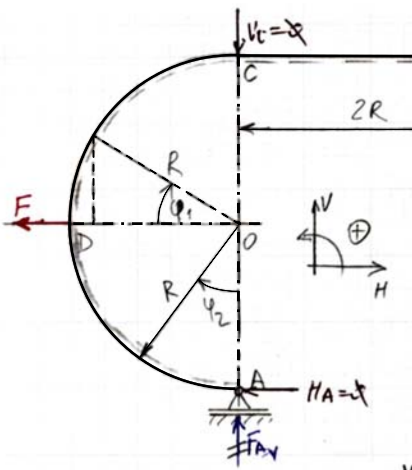


Za statički određeni okvirni nosač zadan i opterećen prema slici a) treba:

- odrediti reakcije veza u osloncima A i B
- odrediti vertikalni pomak u C (w_C = ?)
- odrediti vodoravni pomak oslonca A (u_A = ?)
- odrediti kutni zakret na mjestu oslonca B (α_B = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: F, R, $\mathcal{M} = F\cdot R$, EI_y = konst.

Rješenje:



Jedn. ravnoteže:

1. $\Sigma F_H = 0 \quad F_{BH} = F + H_A \quad \rightarrow F_{BH} = F$

2. $\Sigma F_V = 0 \quad F_A - F_{BV} - V_C = 0$

3. $\Sigma M_B = 0 \quad V_C \cdot 2R + \cancel{\mathcal{M}} + M_B - \cancel{FR} - F_A \cdot 2R - H_A \cdot 2R = 0 \; / : 2R$

$$F_A = 0 + V_C - H_A + \frac{M_B}{2R}, \quad F_{BV} = F_A - V_C = 0 - H_A + \frac{M_B}{2R}$$

Moment savijanja i derivacije:

$$M_x = \mathcal{M} + M_B - F_{BV}\cdot x = F\cdot R + M_B + H_A\cdot x - \frac{M_B}{2R}\cdot x = FR + H_A\cdot x + M_B\left(1 - \frac{x}{2R}\right)$$

$$M_{\varphi_1} = F\cdot R\cdot \sin\varphi_1 + H_A\cdot R(1+\sin\varphi_1) - F_A R\cdot\cos\varphi_1 =$$
$$= FR\cdot\sin\varphi_1 + H_A\cdot R(1+\sin\varphi_1) - V_C\cdot R\cdot\cos\varphi_1 - H_A R\cdot\cos\varphi_1 - \frac{M_B}{2}\cos\varphi_1 =$$
$$= FR\cdot\sin\varphi_1 + H_A\cdot R(1+\sin\varphi_1 - \cos\varphi_1) - V_C R\cdot\cos\varphi_1 - M_B\cdot\frac{\cos\varphi_1}{2}$$

$$M_{\varphi_2} = 0$$

	$\frac{\partial M_i}{\partial V_C}$	$\frac{\partial M_i}{\partial H_A}$	$\frac{\partial M_i}{\partial M_B}$
M_x	0	x	$\left(1-\frac{x}{2R}\right)$
M_{φ_1}	$-R\cdot\cos\varphi_1$	$R(1+\sin\varphi_1 - \cos\varphi_1)$	$-\frac{\cos\varphi_1}{2}$

Vertikalni pomak u C:

$$w_C = \left(\frac{\partial U}{\partial V_C}\right)_{V_C=0} = \frac{F}{EI_y}\int_0^{\frac{\pi}{2}} R\cdot\sin\varphi_1\cdot(-R\cdot\cos\varphi_1)\cdot R\,d\varphi_1 = -\frac{FR^3}{EI_y}\cdot\frac{1}{2} \quad (\uparrow)$$

Vodoravni pomak u A:

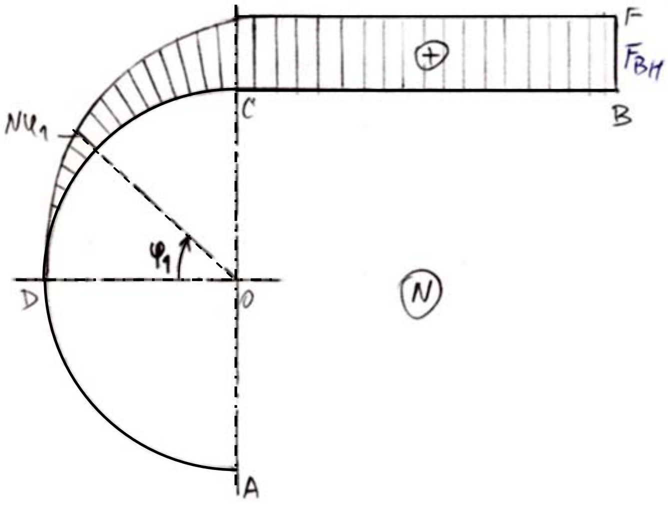
$$u_A = \left(\frac{\partial U}{\partial H_A}\right)_{H_A=0} = \frac{F}{EI_y}\left[\int_0^{2R} R\cdot x\,dx + \int_0^{\pi/2} R\cdot\sin\varphi_1\cdot R(1+\sin\varphi_1 - \cos\varphi_1)R\,d\varphi_1\right] = \frac{FR^3}{EI_y}\left(2 + 1 + \frac{\pi}{4} - \frac{1}{2}\right) =$$
$$= \left(\frac{5}{2} + \frac{\pi}{4}\right)\frac{FR^3}{EI_y} \approx 3{,}2854\cdot\frac{FR^3}{EI_y} \quad (\leftarrow)$$

Kutni zakret u B:

$$\alpha_B = \left(\frac{\partial U}{\partial M_B}\right)_{M_B=0} = \frac{F}{EI_y}\left[\int_0^{2R} R\cdot\left(1-\frac{x}{2R}\right)dx + \int_0^{\pi/2} R\cdot\sin\varphi_1\left(-\frac{\cos\varphi_1}{2}\right)R\,d\varphi_1\right] = \frac{FR^2}{EI_y}\left(2 - \frac{1}{\cancel{2}}\cdot\cancel{2} - \frac{1}{2}\cdot\frac{1}{2}\right) = \frac{3}{4}\frac{FR^2}{EI_y} (\circlearrowleft)$$

Potrebne vrijednosti integrala trigonometrijskih funkcija u ovom primjeru dane su u tablici.

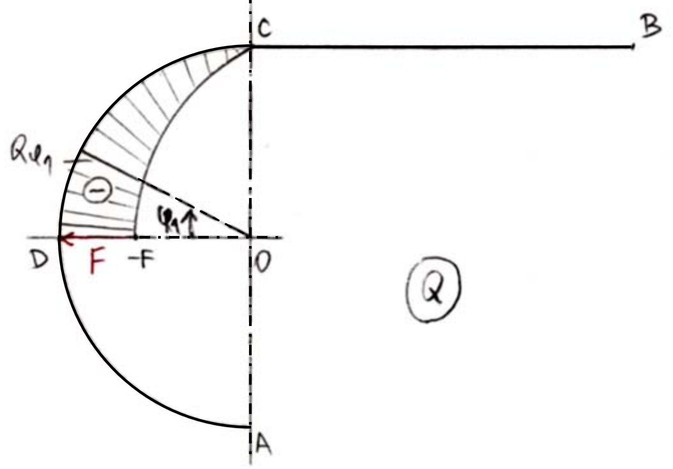
Dijagrami unutarnjih sila duž konture okvirnog nosača:



$N_x = F_{BH} = F$,

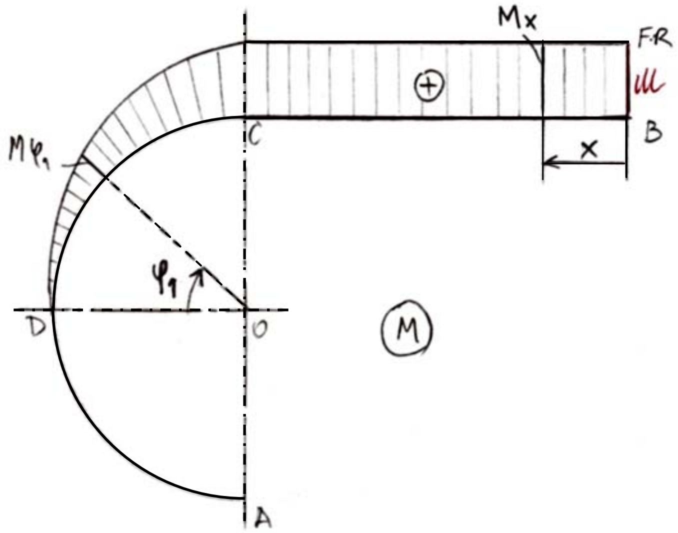
$N_{\varphi_1} = F \cdot \sin\varphi_1$, $0 \le \varphi_1 \le \frac{\pi}{2}$

$N_{\varphi_2} = 0$



$Q_x = 0$, $Q_{\varphi_2} = 0$

$Q_{\varphi_1} = -F \cdot \cos\varphi_1$, $0 \le \varphi_1 \le \frac{\pi}{2}$



$M_x = \mathcal{M} = F \cdot R$, $M_{\varphi_2} = 0$

$M_{\varphi_1} = F \cdot R \cdot \sin\varphi_1$, $0 \le \varphi_1 \le \frac{\pi}{2}$

(U skorijoj budućnosti, primjer će biti iscrtan i ispisan uobičajenom tehnikom, a sada se ovdje daje skeniran iz radnog materijala!).