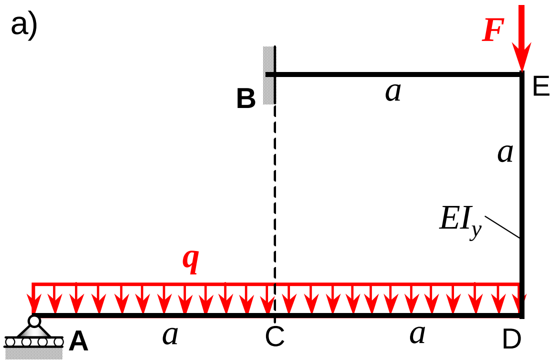
9. Zadatak: Statički neodređeni ravninski okvirni nosač

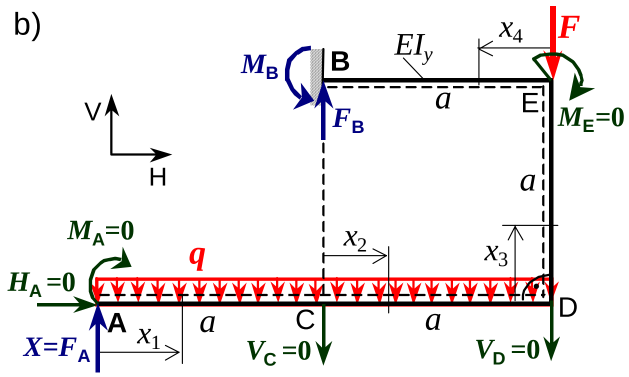


Za statički neodređeni ravninski okvirni nosač ABCDE zadan i opterećen prema slici a) treba:

- odrediti reakcije veza u osloncima A i B
- odrediti vertikalne pomake u točkama C i D ($w_C = ?$, $w_D = ?$)
- vodoravni pomak oslonca A ($u_A = ?$)
- kutne zakrete u točkama A ($\alpha_A = ?$) i E ($\alpha_E = ?$) ,
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: $\boldsymbol{q}$, $\boldsymbol{F = q \cdot a}$, a, $EI_y = \text{konst.}$

Rješenje:



Rješenje:

Jednadžbe ravnoteže za nosač, slika b), jesu:

1. $\sum F_H = 0 \quad F_{BH} = 0$,
2. $\sum F_V = 0 \quad F_A + F_B - F - 2qa = 0$,
3. $\sum M_B = 0 \quad -F_A \cdot a + M_B - F \cdot a = 0 \,/ : a$

Slijedi: $F_B = 3qa - F_A$,

$M_B = F \cdot a + F_A \cdot a$.

Zadatak je jedanput statički neodređen, jer jest: $n = k - s = 4 - 3 = 1$. Za osnovni statički određeni nosač odabran je okvirni konzolni nosač uklanjanjem oslonca u A, slika b), tj. prekobrojna je nepoznata sila: $X = F_A$.

4. Poučak o minimumu energije deformiranja jest ($i = 4$):

$$\frac{\partial U}{\partial X} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_b(x_i)\cdot\frac{\partial M_b(x_i)}{\partial X}\mathrm{d}x_i\right)\right] = 0 \,/ \cdot EI_y$$

Za određivanje vodoravnih i vertikalnih pomaka te kuta zakreta u zadanim točkama okvirnog nosača, prema drugom Castiglianovom poučku potrebno je dodati "nulte sile" u tim točkama. Njihov utjecaj je u derivacijama po tim silama, te su one u momentima savijanja duž konture okvirnog nosača, slika b).

Momenti savijanja duž konture nosača $M_b(x_i) = M_y(x_i)$ i potrebne derivacije jesu:

Momenti savijanja $M_b(x_i)$ dijelova nosača:	$\frac{\partial M_b(x_i)}{\partial X}$	$\frac{\partial M_b(x_i)}{\partial V_C}$	$\frac{\partial M_b(x_i)}{\partial V_D}$	$\frac{\partial M_b(x_i)}{\partial H_A}$	$\frac{\partial M_b(x_i)}{\partial M_A}$	$\frac{\partial M_b(x_i)}{\partial M_E}$
$M_b(x_1) = -X \cdot x_1 + \frac{q \cdot x_1^2}{2} - M_A$	$-x_1$	0	0	0	-1	0
$M_b(x_2) = -X \cdot (a + x_2) + \frac{q(a+x_2)^2}{2} - M_A + V_C \cdot x_2$	$-(a+x_2)$	x_2	0	0	-1	0
$M_b(x_3) = -X \cdot 2a + 2qa^2 - M_A + V_C \cdot a + H_A \cdot x_3$	$-2a$	a	0	x_3	-1	0
$M_b(x_4) = -F \cdot x_4 - X \cdot (2a - x_4) + 2qa \cdot (a - x_4) + V_C \cdot (a - x_4) - V_D \cdot x_4 + H_A \cdot a - M_A - M_E$	$(x_4 - 2a)$	$(a - x_4)$	$-x_4$	a	-1	-1

Uvrštavanjem izraza za momente savijanja $M_{\mathrm{b}}(x_i)$ i derivacije $\dfrac{\partial M_{\mathrm{b}}(x_i)}{\partial X}$ iz tablice u izraz (4), uz dodane "sile" $V_{\mathrm{C}} = V_{\mathrm{D}} = H_{\mathrm{A}} = M_{\mathrm{A}} = M_{\mathrm{E}} = 0$, sređivanjem slijedi vrijednost nepoznate sile $\boldsymbol{X}$ (vertikalna reakcija veze u osloncu A):

$$\int_0^a \left(-X \cdot x_1 + \frac{q \cdot x_1^2}{2} \right) \cdot (-x_1)\,\mathrm{d}x_1 + \int_0^a \left[\left(-X \cdot (a + x_2) + \frac{q \cdot (a + x_2)^2}{2} \right) \right] \cdot \left[-(a + x_2) \right] \mathrm{d}x_2 +$$

$$+ \int_0^a \left(-X \cdot 2a + 2qa^2 \right) \cdot (-2a)\,\mathrm{d}x_3 + \int_0^a \left[-X \cdot (2a - x_4) + 2qa \cdot (a - x_4) - F \cdot x_4 \right] \cdot \left[-(2a - x_4) \right] \mathrm{d}x_4 = 0.$$

Integriranjem i sređivanjem izraza te dijeljenjem s a^3, slijedi:

$$X \cdot 9 = 7 \cdot qa \quad \rightarrow \quad \boxed{\boldsymbol{X} = \frac{7}{9} qa = F_{\mathrm{A}}}.$$

Komponente reakcija veza u osloncu B jesu:

$$F_{\mathrm{B}} = 3qa - F_{\mathrm{A}} = \frac{20}{9} qa\,, \quad M_{\mathrm{B}} = qa^2 + F_{\mathrm{A}} \cdot a = \frac{16}{9} qa^2\,.$$

Za određivanje deformacija okvirnog nosača: vertikalnih pomaka u točkama C i D (E), vodoravnog pomaka oslonca A te kutnih zakreta u A i E, prema drugom Castiglianovom poučku potrebno je dodati "nulte sile" (sile ili momente) u tim točkama. Njihov utjecaj je u derivacijama momenata savijanja po tim silama, te su one izračunate u momentima savijanja duž konture okvirnog nosača, tablica i slika b).

Vertikalni pomak nosača u točki C jest:

$$w_{\mathrm{C}} = \left(\frac{\partial U}{\partial V_{\mathrm{C}}} \right)_{V_{\mathrm{C}}=0} = \frac{1}{EI_y} \left[\sum_{i=1}^{4} \left(\int_0^{l_i} M_{\mathrm{b}}(x_i) \cdot \frac{\partial M_{\mathrm{b}}(x_i)}{\partial V_{\mathrm{C}}} \mathrm{d}x_i \right) \right] = \frac{1}{EI_y} \left\{ \int_0^a \left(-F_{\mathrm{A}} \cdot (a + x_2) + \frac{q \cdot (a + x_2)^2}{2} \right) \cdot x_2\,\mathrm{d}x_2 + \right.$$

$$\left. + \int_0^a \left(-F_{\mathrm{A}} \cdot 2a + 2qa^2 \right) \cdot a\,\mathrm{d}x_3 + \int_0^a \left[-F_{\mathrm{A}} \cdot (2a - x_4) + 2qa \cdot (a - x_4) - F \cdot x_4 \right] \cdot (a - x_4)\,\mathrm{d}x_4 \right\} = \frac{447}{648} \cdot \frac{qa^4}{EI_y} \left(\downarrow \right).$$

Vertikalni pomak nosača u točki D (E) jest:

$$w_{\mathrm{D}} = \left(\frac{\partial U}{\partial V_{\mathrm{D}}} \right)_{V_{\mathrm{D}}=0} = \frac{1}{EI_y} \left[\sum_{i=1}^{4} \left(\int_0^{l_i} M_{\mathrm{b}}(x_i) \cdot \frac{\partial M_{\mathrm{b}}(x_i)}{\partial V_{\mathrm{D}}} \mathrm{d}x_i \right) \right] = \frac{1}{EI_y} \left\{ \int_0^a \left[-F_{\mathrm{A}} \cdot (2a - x_4) + 2qa \cdot (a - x_4) - F \cdot x_4 \right] \cdot \right.$$

$$\left. \cdot (-x_4)\,\mathrm{d}x_4 \right\} = \frac{14}{27} \cdot \frac{qa^4}{EI_y} \cong 0{,}51852 \cdot \frac{qa^4}{EI_y} = w_{\mathrm{E}} \left(\downarrow \right).$$

Vodoravni pomak nosača na mjestu oslonca A jest:

$$u_{\mathrm{A}} = \left(\frac{\partial U}{\partial H_{\mathrm{A}}} \right)_{H_{\mathrm{A}}=0} = \frac{1}{EI_y} \left[\sum_{i=1}^{4} \left(\int_0^{l_i} M_{\mathrm{b}}(x_i) \cdot \frac{\partial M_{\mathrm{b}}(x_i)}{\partial H_{\mathrm{A}}} \mathrm{d}x_i \right) \right] = \frac{1}{EI_y} \left\{ \int_0^a \left(-F_{\mathrm{A}} \cdot 2a + 2qa^2 \right) \cdot x_3\,\mathrm{d}x_3 + \right.$$

$$\left. + \int_0^a \left[-F_{\mathrm{A}} \cdot (2a - x_4) + 2qa \cdot (a - x_4) - F \cdot x_4 \right] \cdot a\,\mathrm{d}x_4 \right\} = -\frac{4}{9} \cdot \frac{qa^4}{EI_y} \cong -0{,}444 \cdot \frac{qa^4}{EI_y} = u_{\mathrm{D}} \quad \left(\leftarrow \right).$$

Kutni zakret nosača na mjestu oslonca A jest:

$$\alpha_{\mathrm{A}} = \left(\frac{\partial U}{\partial M_{\mathrm{A}}} \right)_{M_{\mathrm{A}}=0} = \frac{1}{EI_y} \left[\sum_{i=1}^{4} \left(\int_0^{l_i} M_{\mathrm{b}}(x_i) \cdot \frac{\partial M_{\mathrm{b}}(x_i)}{\partial M_{\mathrm{A}}} \mathrm{d}x_i \right) \right] = \frac{1}{EI_y} \left\{ \int_0^a \left(-F_{\mathrm{A}} \cdot x_1 + \frac{q \cdot x_1^2}{2} \right) \cdot (-1)\,\mathrm{d}x_1 + \right.$$

$$+\int_0^a\left(-F_{\mathrm{A}}\cdot(a+x_2)+\frac{q\cdot(a+x_2)^2}{2}\right)\cdot(-1)\,\mathrm{d}x_2+\int_0^a\frac{4}{9}qa^2\cdot(-1)\,\mathrm{d}x_3+\int_0^a\left[-F_{\mathrm{A}}\cdot(2a-x_4)+2qa\cdot(a-x_4)-F\cdot x_4\right]\cdot$$

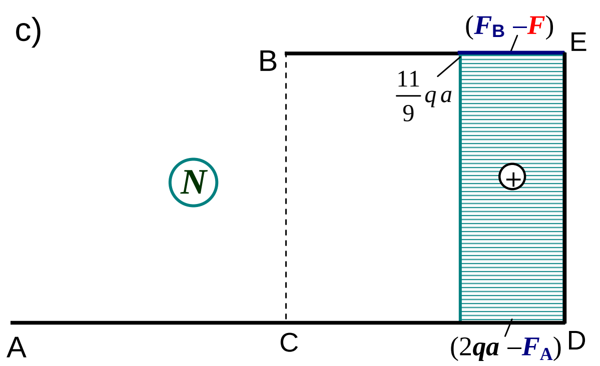
$$\cdot(-1)\,\mathrm{d}x_4\Big\}=\frac{4}{9}\cdot\frac{qa^3}{EI_y}\cong 0{,}444\cdot\frac{qa^3}{EI_y}\ \ (\circlearrowright).$$

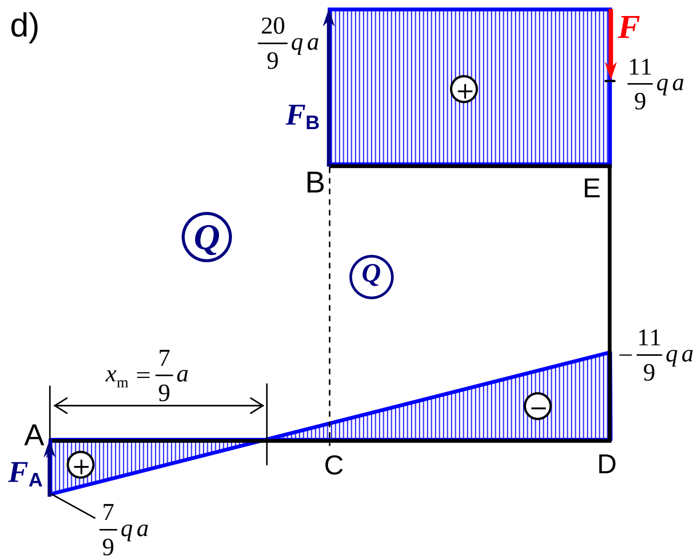
Kutni zakret nosača u točki E jest:

$$\alpha_{\mathrm{E}}=\left(\frac{\partial U}{\partial M_{\mathrm{E}}}\right)_{M_{\mathrm{E}}=0}=\frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i}M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial M_{\mathrm{E}}}\mathrm{d}x_i\right)\right]=\frac{1}{EI_y}\Big\{\int_0^a\left[-F_{\mathrm{A}}\cdot(2a-x_4)+2qa\cdot(a-x_4)-F\cdot x_4\right]\cdot$$

$$\cdot(-1)\,\mathrm{d}x_4\Big\}=\frac{2}{3}\cdot\frac{qa^3}{EI_y}\cong 0{,}667\cdot\frac{qa^3}{EI_y}\ \ (\circlearrowright).$$

Dijagrami unutarnjih uzdužnih i poprečnih sila duž konture okvirnog nosača:





Momenti savijanja u karakterističnim točkama okvirnog nosača jesu:

$$M_{\mathrm{bA}}=0,\ M_{\mathrm{bB}}=-\frac{16}{9}qa^2,\ M_{\mathrm{bE}}=F_{\mathrm{B}}\cdot a-M_{\mathrm{B}}=\frac{4}{9}qa^2=M_{\mathrm{bD}}.$$

$$Q(x_{\mathrm{m}})=F_{\mathrm{A}}-q\cdot x_{\mathrm{m}}=0\rightarrow x_m=\frac{F_{\mathrm{A}}}{q}=\frac{7}{9}a.\ \ M_{\mathrm{b}}(x_{\mathrm{m}})=-F_{\mathrm{A}}\cdot x_{\mathrm{m}}+\frac{q\cdot x_{\mathrm{m}}^2}{2}=-\frac{49}{162}qa^2\cong-0{,}3025qa^2.$$

Dijagram momenata savijanja duž konture nosača i elastična linija okvirnog nosača: