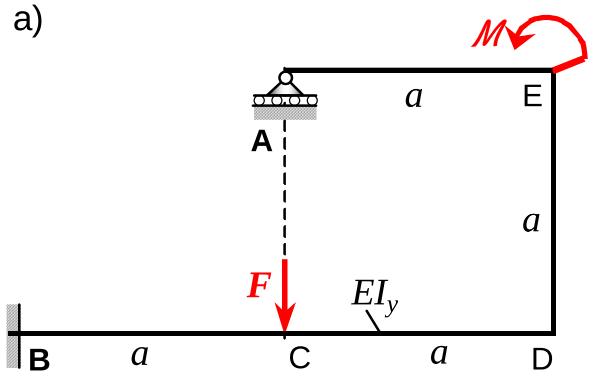
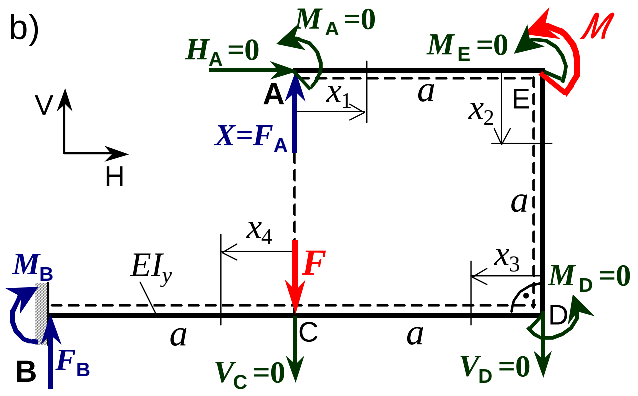
8. Zadatak: Statički neodređeni ravninski okvirni nosač



Za statički neodređeni ravninski okvirni nosač ABCDE zadan i opterećen prema slici a) treba:
- odrediti reakcije veza u osloncima A i B
- odrediti vertikalne pomake u točkama C i D (w_C = ?, w_D = ?)
- vodoravni pomak oslonca A (u_A = ?)
- kutne zakrete u točkama A i C (α_A, α_C = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: $\boldsymbol{F}$, a, $\mathcal{M} = F \cdot a$, EI_y = konst.



Rješenje:

Jednadžbe ravnoteže za okvirni nosač, slika b), jesu:

1. $\sum F_H = 0 \quad F_{BH} = 0$,
2. $\sum F_V = 0 \quad -F + F_A + F_B = 0$,
3. $\sum M_B = 0 \quad -M_B + F_A \cdot a - F \cdot a + \mathcal{M} = 0$.

Slijedi: $F_B = F - F_A$,

$$M_B = F_A \cdot a - F \cdot a + \mathcal{M} = F_A \cdot a.$$

Zadatak je jedanput statički neodređen, jer jest: $n = k - s = 4 - 3 = 1$. Za osnovni statički određeni nosač odabran je okvirni konzolni nosač uklanjanjem oslonca u A, slika b), tj. prekobrojna je nepoznata sila: $\boldsymbol{X} = \boldsymbol{F}_A$.

4. Poučak o minimumu energije deformiranja jest ($i = 4$):

$$\frac{\partial U}{\partial X} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_b(x_i)\cdot\frac{\partial M_b(x_i)}{\partial X}\mathrm{d}x_i\right)\right] = 0 \,/\cdot EI_y$$

Za određivanje vodoravnih i vertikalnih pomaka te kuta zakreta u zadanim točkama okvirnog nosača, prema drugom Castiglianovom poučku potrebno je dodati ″nulte sile″ u tim točkama. Njihov utjecaj je u derivacijama po tim silama, te su one u momentima savijanja duž konture okvirnog nosača, slika b).

Momenti savijanja duž konture nosača $M_b(x_i) = M_y(x_i)$ i potrebne derivacije jesu:

Momenti savijanja $M_b(x_i)$ dijelova okvirnog nosača:	$\frac{\partial M_b(x_i)}{\partial X}$	$\frac{\partial M_b(x_i)}{\partial V_C}$	$\frac{\partial M_b(x_i)}{\partial V_D}$	$\frac{\partial M_b(x_i)}{\partial H_A}$	$\frac{\partial M_b(x_i)}{\partial M_A}$	$\frac{\partial M_b(x_i)}{\partial M_E}$	$\frac{\partial M_b(x_i)}{\partial M_D}$
$M_b(x_1) = X \cdot x_1 - M_A$	x_1	0	0	0	-1	0	0
$M_b(x_2) = X \cdot a - \mathcal{M} - M_A + H_A \cdot x_2 - M_E$	a	0	0	x_2	-1	-1	0
$M_b(x_3) = X \cdot (a - x_3) - \mathcal{M} - M_A +$ $+ H_A \cdot a - M_E + V_D \cdot x_3 - M_D$	$(a - x_3)$	0	x_3	a	-1	-1	-1
$M_b(x_4) = -X \cdot x_4 + F \cdot x_4 - \mathcal{M} + H_A \cdot a -$ $- M_A - M_E + V_C \cdot x_4 - M_D + V_D \cdot (a + x_4)$	$-x_4$	x_4	$(a + x_4)$	a	-1	-1	-1

Moment savijanja $M_{\mathrm{b}}(x_i)=M_y(x_i)$ uzet je pozitivan, ako on na strani dijela nosača označenom crtkanom linijom, slika b), izaziva rastezna (vlačna) naprezanja.

Uvrštavanjem izraza za momente savijanja $M_{\mathrm{b}}(x_i)$ i derivacije $\dfrac{\partial M_{\mathrm{b}}(x_i)}{\partial X}$ iz tablice u izraz (4), uz dodane "sile" $V_{\mathrm{C}}=V_{\mathrm{D}}=H_{\mathrm{A}}=M_{\mathrm{A}}=M_{\mathrm{D}}=M_{\mathrm{E}}=0$, sređivanjem slijedi vrijednost nepoznate sile $\boldsymbol{X}$ (vertikalna reakcija veze u osloncu A):

$$\int_0^a (X\cdot x_1)\cdot x_1\,\mathrm{d}x_1+\int_0^a (X\cdot a-F\cdot a)\cdot a\,\mathrm{d}x_2+\int_0^a [X\cdot(a-x_3)-F\cdot a]\cdot(a-x_3)\,\mathrm{d}x_3+$$
$$+\int_0^a [-X\cdot x_4-F\cdot(a-x_4)]\cdot(-x_4)\,\mathrm{d}x_4=0.$$

Integriranjem i sređivanjem izraza te dijeljenjem s a^3, slijedi:

$$2X-\frac{4}{3}\cdot F=0\quad\rightarrow\quad\boxed{X=\frac{2}{3}F=F_{\mathrm{A}}}.$$

Komponente reakcije veza u osloncu B jesu:

$$F_{\mathrm{B}}=F-F_{\mathrm{A}}=\frac{1}{3}F,\quad M_{\mathrm{B}}=F_{\mathrm{A}}\cdot a=\frac{2}{3}F\,a\,.$$

Sređeni momenti savijanja duž konture okvirnog nosača jesu:

$$M_{\mathrm{b}}(x_1)=F_{\mathrm{A}}\cdot x_1=\frac{2}{3}F\cdot x_1\,,$$

$$M_{\mathrm{b}}(x_2)=F_{\mathrm{A}}\cdot a-\mathcal{M}=\frac{1}{3}F\cdot a\,,$$

$$M_{\mathrm{b}}(x_3)=F_{\mathrm{A}}\cdot(a-x_3)-\mathcal{M}=-\frac{F}{3}\cdot(a+2x_3)\,,$$

$$M_{\mathrm{b}}(x_4)=-F_{\mathrm{A}}\cdot x_4+F\cdot x_4-\mathcal{M}=\frac{F}{3}\cdot(x_4-3a)\,.$$

Za određivanje deformacija okvirnog nosača: vertikalnih pomaka u točkama C i D (i E), vodoravnog pomaka oslonca A (i u točki E) te kutnog zakreta u A, E i D, prema drugom Castiglianovom poučku potrebno je dodati ″nulte sile" (sile ili momente) u tim točkama. Njihov utjecaj je u derivacijama momenata savijanja po tim silama, te su one izračunate u momentima savijanja duž konture okvirnog nosača, tablica i slika b).

Vertikalni pomak nosača u točki C jest:

$$w_{\mathrm{C}}=\left(\frac{\partial U}{\partial V_{\mathrm{C}}}\right)_{V_{\mathrm{C}}=0}=\frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial V_{\mathrm{C}}}\mathrm{d}x_i\right)\right]=\frac{F}{3EI_y}\left[\int_0^a (x_4-3a)\cdot x_4\,\mathrm{d}x_4\right]=-\frac{7}{18}\cdot\frac{Fa^3}{EI_y}\;(\uparrow)\,.$$

Vertikalni pomak nosača u točki D (i E) jest:

$$w_{\mathrm{D}}=\left(\frac{\partial U}{\partial V_{\mathrm{D}}}\right)_{V_{\mathrm{D}}=0}=\frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial V_{\mathrm{D}}}\mathrm{d}x_i\right)\right]=\frac{F}{3EI_y}\left\{\int_0^a (-a-2x_3)\cdot x_3\,\mathrm{d}x_3+\right.$$
$$\left.+\int_0^a (x_4-3a)\cdot(a+x_4)\,\mathrm{d}x_4\right\}=-\frac{29}{18}\cdot\frac{Fa^3}{EI_y}=-1{,}611\cdot\frac{Fa^3}{EI_y}=w_{\mathrm{E}}\;(\uparrow).$$

Vodoravni pomak nosača na mjestu oslonca A (u C i D) jest:

$$u_{\mathrm{A}} = \left(\frac{\partial U}{\partial H_{\mathrm{A}}}\right)_{H_{\mathrm{A}}=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial H_{\mathrm{A}}}\mathrm{d}x_i\right)\right] = \frac{F}{3EI_y}\left[\int_0^a a\cdot x_2\,\mathrm{d}x_2 - \int_0^a (a+2x_3)\cdot a\,\mathrm{d}x_3 + \right.$$

$$\left. + \int_0^a (x_4-3a)\cdot a\,\mathrm{d}x_4\right] = -\frac{5}{3}\cdot\frac{Fa^3}{EI_y} = u_{\mathrm{E}} \quad (\leftarrow).$$

Kutni zakret nosača na mjestu oslonca A jest:

$$\alpha_{\mathrm{A}} = \left(\frac{\partial U}{\partial M_{\mathrm{A}}}\right)_{M_{\mathrm{A}}=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial M_{\mathrm{A}}}\mathrm{d}x_i\right)\right] = \frac{F}{3EI_y}\left\{\int_0^a 2x_1\cdot(-1)\,\mathrm{d}x_1 + \int_0^a a\cdot(-1)\,\mathrm{d}x_2 - \right.$$

$$\left. -\int_0^a (a+2x_3)\cdot(-1)\,\mathrm{d}x_3 + \int_0^a (x_4-3a)\cdot(-1)\,\mathrm{d}x_4\right\} = \frac{1}{6}\cdot\frac{Fa^2}{EI_y} \quad (\circlearrowleft).$$

Kutni zakret nosača u točki D jest:

$$\alpha_{\mathrm{D}} = \left(\frac{\partial U}{\partial M_{\mathrm{D}}}\right)_{M_{\mathrm{D}}=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial M_{\mathrm{D}}}\mathrm{d}x_i\right)\right] = \frac{F}{3EI_y}\left\{-\int_0^a (a+2x_3)\cdot(-1)\,\mathrm{d}x_3 + \right.$$

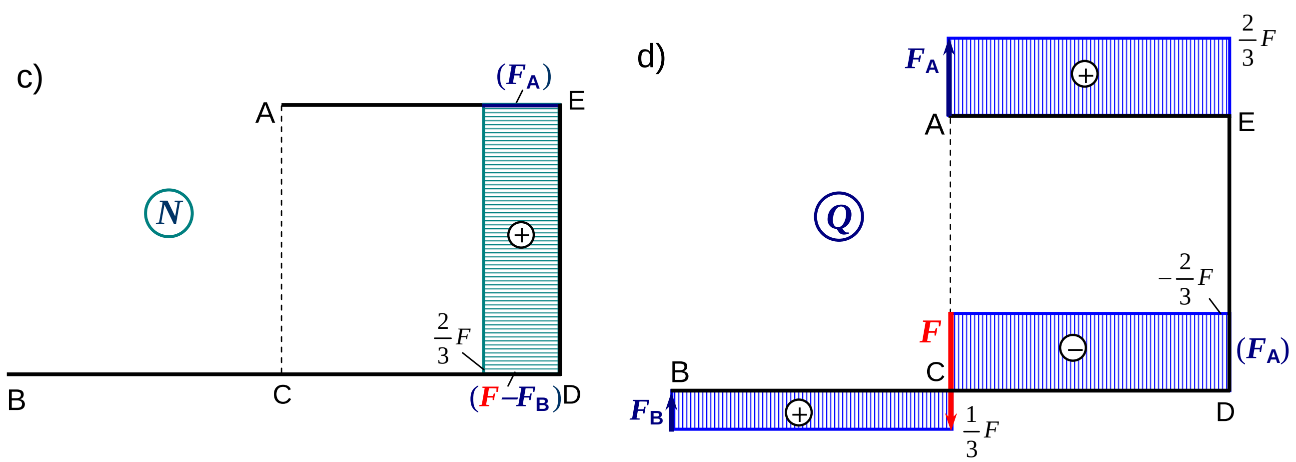
$$\left. + \int_0^a (x_4-3a)\cdot(-1)\,\mathrm{d}x_4\right\} = \frac{3}{2}\cdot\frac{Fa^2}{EI_y} \quad (\circlearrowleft).$$

Kutni zakret nosača u točki E jest:

$$\alpha_{\mathrm{E}} = \left(\frac{\partial U}{\partial M_{\mathrm{E}}}\right)_{M_{\mathrm{E}}=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial M_{\mathrm{E}}}\mathrm{d}x_i\right)\right] = \frac{F}{3EI_y}\left\{\int_0^a a\cdot(-1)\,\mathrm{d}x_2 - \int_0^a (a+2x_3)\cdot(-1)\,\mathrm{d}x_3 - \right.$$

$$\left. + \int_0^a (x_4-3a)\cdot(-1)\,\mathrm{d}x_4\right\} = \frac{1}{2}\cdot\frac{Fa^2}{EI_y} \quad (\circlearrowleft).$$

Dijagrami unutarnjih uzdužnih i poprečnih sila duž konture okvirnog nosača:



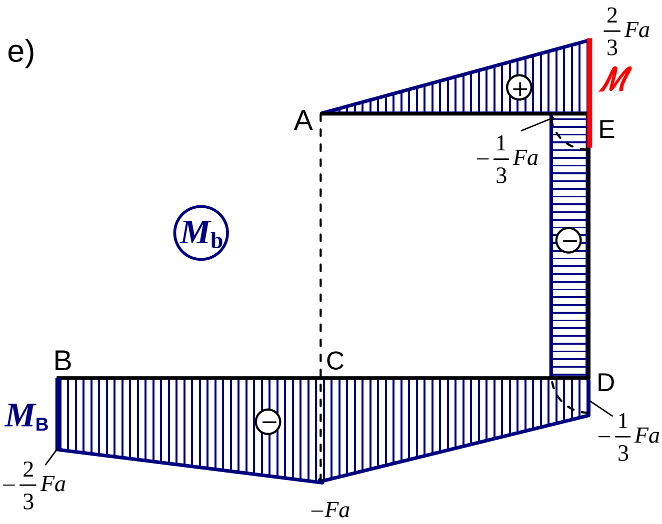
Momenti savijanja u karakterističnim točkama okvirnog nosača jesu:

$$M_{\mathrm{bA}} = 0, \quad M_{\mathrm{bB}} = -\frac{2}{3}F\,a, \quad (M_{\mathrm{bE}})_{\mathrm{L}} = F_{\mathrm{A}}\cdot a = \frac{2}{3}F\,a, \quad (M_{\mathrm{bE}})_{\mathrm{D}} = (M_{\mathrm{bE}})_{\mathrm{L}} - \mathcal{M} = -\frac{1}{3}F\,a = M_{\mathrm{bD}},$$

$$M_{\mathrm{bC}} = -\mathcal{M} = -F\,a.$$

Dijagram momenata savijanja duž konture nosača i elastična linija okvirnog nosača:

e)



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