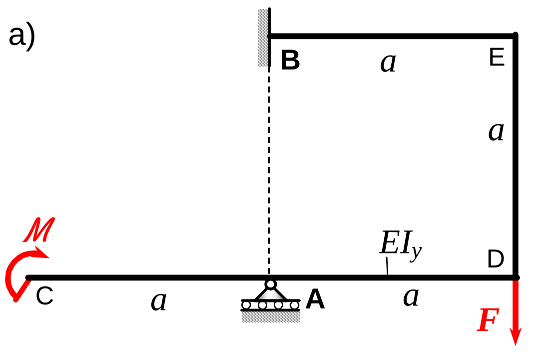
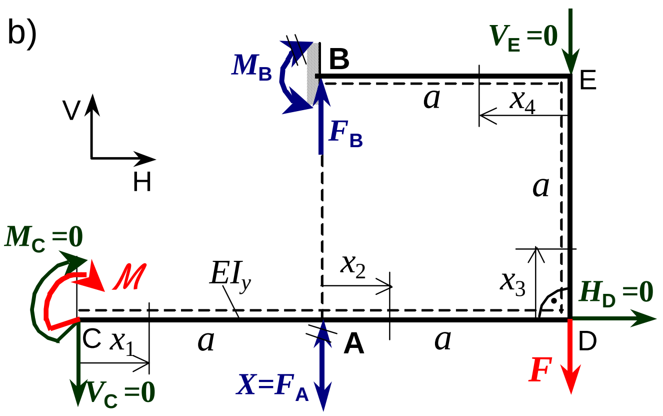
7. Zadatak: Statički neodređeni ravninski okvirni nosač



Za statički neodređeni ravninski okvirni nosač ABCDE zadan i opterećen prema slici a) treba:

- odrediti reakcije veza u osloncima A i B
- odrediti vertikalne pomake u točkama C i E (w_C = ?, w_E = ?)
- vodoravni pomak u točki D (u_D = ?)
- kutni zakret u točki C (α_C = ?)
- skicirati i kotirati dijagrame uzdužnih i poprečnih sila te momenta savijanja duž konture nosača.

Zadano: $\boldsymbol{F}$, a, $\mathcal{M} = F \cdot a$, $EI_y = \text{konst.}$



Rješenje:

Jednadžbe ravnoteže za okvirni nosač, slika b), jesu:

1. $\sum F_H = 0 \quad F_{BH} = 0$,
2. $\sum F_V = 0 \quad -F + F_A + F_B = 0$,
3. $\sum M_B = 0 \quad -M_B - F \cdot a - \mathcal{M} = 0$.

Slijedi: $F_B = F - F_A$,

$$M_B = -F \cdot a - \mathcal{M} = -2F \cdot a .$$

Zadatak je jedanput statički neodređen, jer jest: $n = k - s = 4 - 3 = 1$. Za osnovni statički određeni nosač odabran je okvirni konzolni nosač uklanjanjem oslonca u A, slika b), tj. prekobrojna je nepoznata sila: $\boldsymbol{X} = \boldsymbol{F}_A$.

4. Poučak o minimumu energije deformiranja jest ($i = 4$):

$$\frac{\partial U}{\partial X} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_b(x_i)\cdot\frac{\partial M_b(x_i)}{\partial X}\mathrm{d}x_i\right)\right] = 0 \,/\cdot EI_y$$

Za određivanje vodoravnih i vertikalnih pomaka te kuta zakreta u zadanim točkama okvirnog nosača, prema drugom Castiglianovom poučku potrebno je dodati ″nulte sile″ u tim točkama. Njihov utjecaj je u derivacijama po tim silama, te su one u momentima savijanja duž konture okvirnog nosača, slika b).

Momenti savijanja duž konture nosača $M_b(x_i) = M_y(x_i)$ i potrebne derivacije jesu:

Momenti savijanja $M_b(x_i)$ dijelova nosača:	$\frac{\partial M_b(x_i)}{\partial X}$	$\frac{\partial M_b(x_i)}{\partial V_C}$	$\frac{\partial M_b(x_i)}{\partial V_E}$	$\frac{\partial M_b(x_i)}{\partial H_D}$	$\frac{\partial M_b(x_i)}{\partial M_C}$
$M_b(x_1) = -\mathcal{M} + V_C \cdot x_1 - M_C$	0	x_1	0	0	-1
$M_b(x_2) = -\mathcal{M} - X \cdot x_2 + V_C \cdot (a + x_2) - M_C$	$-x_2$	$(a + x_2)$	0	0	-1
$M_b(x_3) = -\mathcal{M} - X \cdot a + V_C \cdot 2a - M_C + H_D \cdot x_3$	a	$2a$	0	x_3	-1
$M_b(x_4) = -\mathcal{M} - X \cdot (a - x_4) - F \cdot x_4 + H_D \cdot a + V_C \cdot (2a - x_4) - M_C - V_E \cdot x_4$	$(x_4 - a)$	$(2a - x_4)$	$-x_4$	a	-1

Uvrštavanjem izraza za momente savijanja $M_{\mathrm{b}}(x_i)$ i derivacije $\dfrac{\partial M_{\mathrm{b}}(x_i)}{\partial X}$ iz tablice u izraz (4), uz dodane "sile" $V_{\mathrm{C}} = V_{\mathrm{D}} = H_{\mathrm{D}} = M_{\mathrm{C}} = 0$, sređivanjem slijedi vrijednost nepoznate sile $\boldsymbol{X}$ (vertikalna reakcija veze u osloncu A):

$$\int_0^a (-X\cdot x_2 - F\cdot a)\cdot(-x_2)\,\mathrm{d}x_2 + \int_0^a (-F\cdot a - X\cdot a)\cdot(-a)\,\mathrm{d}x_3 +$$

$$+\int_0^a \left[-X\cdot(a-x_4) - F\cdot x_4 - F\cdot a\right]\cdot(x_4 - a)\,\mathrm{d}x_4 = 0.$$

Integriranjem i sređivanjem izraza te dijeljenjem s a^3, slijedi:

$$\frac{5}{3}X + \frac{13}{6}\cdot F = 0 \quad \rightarrow \quad \boxed{X = -\frac{13}{10}F = F_{\mathrm{A}} \quad (\downarrow)}.$$

Komponente reakcije veza u osloncu B jesu:

$$F_{\mathrm{B}} = F - F_{\mathrm{A}} = \frac{23}{10}F \ (\uparrow), \quad M_{\mathrm{B}} = -2Fa \ (\circlearrowleft).$$

Sređeni momenti savijanja duž konture okvirnog nosača jesu:

$$M_{\mathrm{b}}(x_1) = -\mathcal{M} = -F\cdot a,$$

$$M_{\mathrm{b}}(x_2) = -\mathcal{M} - F_{\mathrm{A}}\cdot x_2 = F\cdot\left(\frac{13}{10}x_2 - a\right),$$

$$M_{\mathrm{b}}(x_3) = -\mathcal{M} - F_{\mathrm{A}}\cdot a = \frac{3}{10}F\cdot a,$$

$$M_{\mathrm{b}}(x_4) = \mathcal{M} - F\cdot x_4 - F_{\mathrm{A}}\cdot(a - x_4) = \frac{F}{10}\cdot(3a - 23x_4).$$

Za određivanje deformacija okvirnog nosača: vertikalnih pomaka u točkama C i E (i D), vodoravnog pomaka u D (u C i oslonca A) te kutnog zakreta u C, prema drugom Castiglianovom poučku potrebno je dodati ″nulte sile" (sile ili momente) u tim točkama. Njihov utjecaj je u derivacijama momenata savijanja po tim silama, te su one izračunate u momentima savijanja duž konture okvirnog nosača, tablica i slika b).

Vertikalni pomak nosača u točki C jest:

$$w_{\mathrm{C}} = \left(\frac{\partial U}{\partial V_{\mathrm{C}}}\right)_{V_{\mathrm{C}}=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial V_{\mathrm{C}}}\mathrm{d}x_i\right)\right] = -\frac{F}{EI_y}\left\{\int_0^a a\cdot x_1\,\mathrm{d}x_1 + \int_0^a\left(a - \frac{13}{10}x_2\right)\cdot(a + x_2)\,\mathrm{d}x_2 +\right.$$

$$\left. + \int_0^a\left(-\frac{3}{10}a\right)\cdot 2a\,\mathrm{d}x_3 + \frac{1}{10}\cdot\int_0^a(23x_4 - 3a)\cdot(2a - x_4)\,\mathrm{d}x_4\right\} = -\frac{7}{5}\cdot\frac{Fa^3}{EI_y} \ (\uparrow).$$

Vertikalni pomak nosača u točki E (i D) jest:

$$w_{\mathrm{E}} = \left(\frac{\partial U}{\partial V_{\mathrm{E}}}\right)_{V_{\mathrm{E}}=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial V_{\mathrm{E}}}\mathrm{d}x_i\right)\right] = \frac{F}{EI_y}\cdot\frac{1}{10}\cdot\int_0^a(3a - 23x_4)\cdot(-x_4)\,\mathrm{d}x_4 = \frac{37}{60}\cdot\frac{Fa^3}{EI_y} = w_{\mathrm{D}} \ (\downarrow).$$

Vodoravni pomak nosača u D (u C i na mjestu oslonca A) jest:

$$u_{\mathrm{D}} = \left(\frac{\partial U}{\partial H_{\mathrm{D}}}\right)_{H_{\mathrm{D}}=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_{\mathrm{b}}(x_i)\cdot\frac{\partial M_{\mathrm{b}}(x_i)}{\partial H_{\mathrm{D}}}\mathrm{d}x_i\right)\right] = \frac{F}{EI_y}\cdot\frac{1}{10}\left\{\int_0^a(3a)\cdot x_3\,\mathrm{d}x_3 + \int_0^a(3a - 23x_4)\cdot a\,\mathrm{d}x_4\right\} =$$

$$= -\frac{7}{10}\cdot\frac{Fa^3}{EI_y} = u_A = u_C \quad (\leftarrow).$$

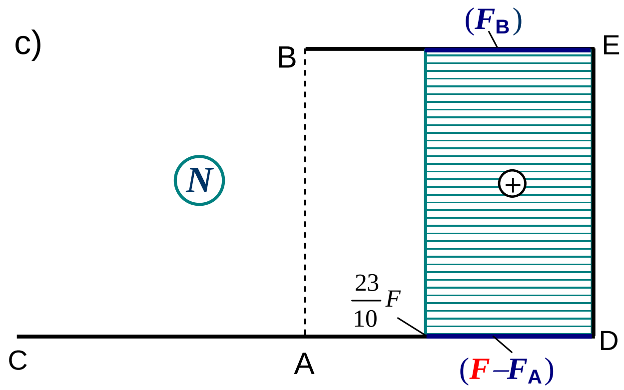
Kutni zakret nosača na mjestu C jest:

$$\alpha_C = \left(\frac{\partial U}{\partial M_C}\right)_{M_C=0} = \frac{1}{EI_y}\left[\sum_{i=1}^{4}\left(\int_0^{l_i} M_b(x_i)\cdot\frac{\partial M_b(x_i)}{\partial M_C}\,dx_i\right)\right] = \frac{F}{EI_y}\left\{\int_0^a(-a)\cdot(-1)\cdot dx_1 + \int_0^a\left(\frac{13}{10}x_2 - a\right)\cdot(-1)\cdot dx_2 + \right.$$

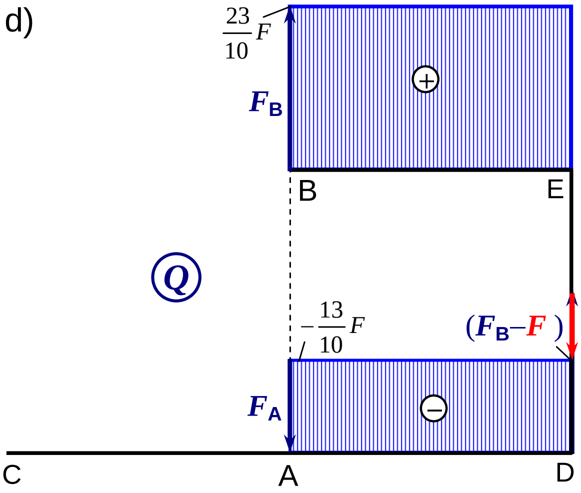
$$\left. + \frac{1}{10}\cdot\int_0^a(3a)\cdot(-1)\cdot dx_3 + \frac{1}{10}\cdot\int_0^a(3a-23x_4)\cdot(-1)\cdot dx_4\right\} = \frac{19}{10}\cdot\frac{Fa^2}{EI_y} \quad (\circlearrowright).$$

Dijagrami unutarnjih uzdužnih i poprečnih sila duž konture okvirnog nosača:

c)



d)

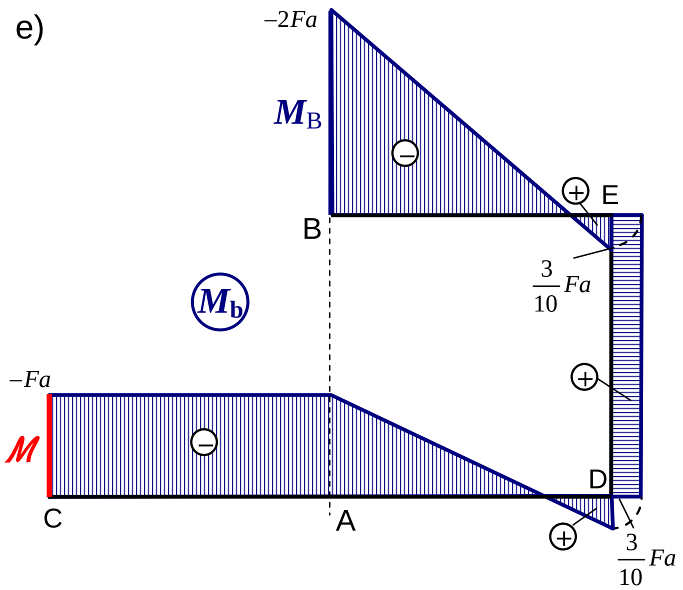


Momenti savijanja u karakterističnim točkama okvirnog nosača jesu:

$$M_{bA} = M_{bC} = -\mathcal{M} = -F\,a,\quad M_{bB} = M_B = -2F\,a,\quad M_{bD} = -\mathcal{M} + F_A\cdot a = \frac{3}{10}F\,a = M_{bE}.$$

Dijagram momenata savijanja duž konture nosača i elastična linija okvirnog nosača:

e)



f)